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Mathematical Model of Automated Underground Parking Garage

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Summary:

This article is based on the kinematics model and reinforcement learning of Ackermann steering geometry. This paper studies the situation of reversing self-driving cars into underground garages aimed at exploring the analysis and modeling of the shortest path and constraint conditions, and further repeated demonstrations based on the Ackerman steering principle, and the operation and verification of simulation, which fully verified the feasibility and practicability of this model.

Keywords: **Ackermann geometric steering, visual positioning, automatic parking, path planning, shortest path**

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1. Background Research

Autonomous driving is currently the most popular direction in the field of artificial intelligence research. However, what is autonomous driving? The closest definition is: A self-driving car, also known as a robot car, autonomous car, or driverless car, is a vehicle that is capable of sensing its environment and moving with little or no human input.

It is an intelligent car robot that can carry people. It achieves high-efficiency and high-reliability driving capabilities through rich perception systems with intelligent behavior systems.

According to KPMG, an authoritative information agency in 2017, it predicts the three most important areas of the three major travel trends in the future. The first is shared travel services provide by Giants like Didi and Uber. The second area is new energy vehicles, where we can find Tesla Company founded by Musk. The third area is autonomous driving. Among all of them, autonomous driving has the highest technical requirements and most influence.

Why is autonomous driving so popular? Let's talk about the problems that can be solved by autonomous driving.

First, it improves transportation efficiency as autonomous driving is a design by engineers based on very experienced old drivers complying with traffic regulations. Large numbers of autonomous driving are deployed and road traffic is smoother, car congestion is reduced, traffic accidents are reduced, the problem of invisible intersections is also solved, and the driver reacts when the traffic light changes. Different speeds may cause secondary congestion. Autonomous driving has increased traffic efficiency by 45% and reduced commuting time.

Second, the vehicle utilization rate has increased as parking spaces in big cities will never be enough. However, no parking spaces are need and point-to-point traffic is reduce, shared transportation services will maximize the popularity as the cost can be reduced.

Third, the most important thing is safety as more than 1 million people die in car accidents every year. 93% of car accidents are caused by humans: drunk driving, fatigued driving, distracted drivers. Humans need to learn from scratch every time, there are always novice drivers on the road, making mistakes over and over again, and self-driving cars can avoid this situation.

As a disruptive innovation, autonomous driving will bring tremendous changes to human travel, urban layout, production, and life, and all stakeholders including governments, companies, and individuals will inevitably participate in their own ways. Among them, witness this great process.

And automatic parking technology is the core technology to solve the last mile of automatic driving, and it is an important part of unmanned driving technology. Automatic parking is the most likely to be the first to land in the automatic driving process. The technical reasons behind it are 1. Autonomous driving in a low-speed environment (less than 5km/h), the self-driving car has enough time to make perception and decision-making; 2. The scene is relatively fixed and specific, usually in the parking lot area, without complicated pedestrian flow. Therefore, solving the last mile unmanned driving is a vane for unmanned driving to achieve breakthroughs.

2. Introduction

2.1 Introduction to the origin of Ackermann Steering Geometry

Ackermann steering is a modern car steering method. When the car turns, the angles of the inner and outer wheels are different, and the turning radius of the inner tire is smaller than the outer tire. Figure 2.1 below is the ideal Ackermann steering.

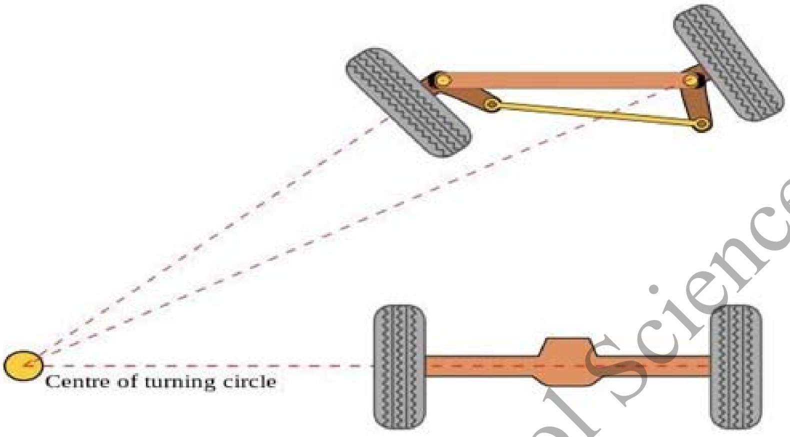


Figure 2.1 Ackermann steering

This steering method was first proposed by Georg Lankensperger, a German carriage engineer, in 1817. His agent Rudolph Ackermann applied for a patent in the United Kingdom in 1818, so this steering principle will be called Ackermann steering geometry from now on.

There are several problems with this shift:

(1) Because it needs to turn around a single axis in the middle, there is a lot of room for swinging, and the front wheel cannot be made large so that the front wheel will easily be lifted when it hits a stone or other obstacle on the road.

(2) Because the two front wheels are parallel, it is easy for the four wheels to form a triangle in the same plane when turning over the head. In this case, the car is the most "stable" and will simply freeze.

In this context, a new trapezoidal (originally rectangular) steering mechanism was proposed. The advantage of this steering mechanism is that the left and right wheels can rotate around independent rotating shafts (kingpins), which greatly reduces the space required for wheel steering is reduced.

There are three types of designs surrounding the Ackerman steering trapezoid, as shown in Figure 2.2:

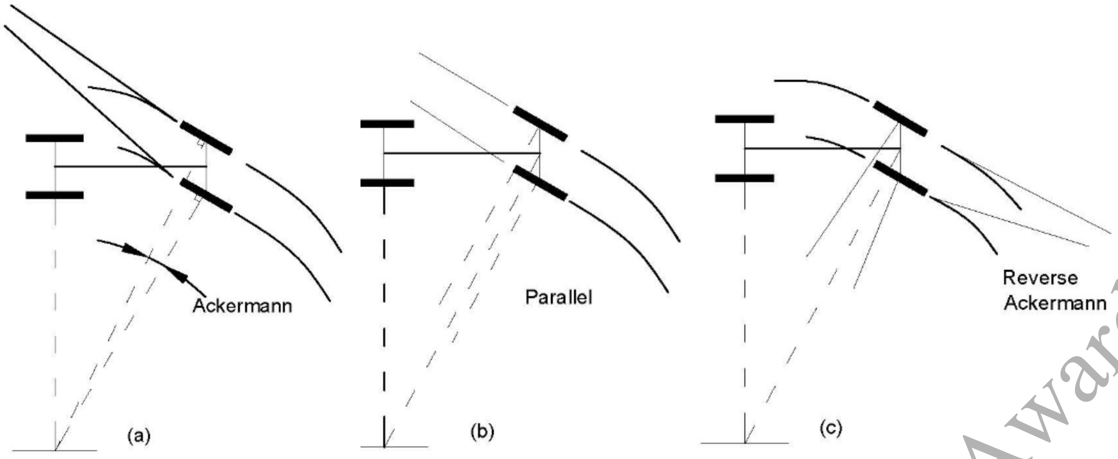


Figure 2.2 Ackermann steering trapezoid

The first type, Ackermann steering trapezoid (picture (a) below), inner wheel angle>outer wheel; ideal Ackermann trapezoid inner and outer wheel angle perpendicular to the rear axle extension line

The second type, parallel steering trapezoid (picture (b) below), inner wheel angle = outer wheel

The third type, anti-Ackermann steering trapezoid (picture (c) below), inner wheel angle <outer wheel

In the case of reversing and entering the garage discussed in this article, the car is turning at a low speed, the lateral acceleration is small, which can be approximated to 0, which can be considered to meet the first category.

When the vehicle is turning at low speed, the lateral acceleration is very small (think 0). The ideal steering method is shown in Figure (a), which is also called ideal Ackermann steering.

When the vehicle turns at high speed, due to the large lateral angular velocity (which cannot be ignored), according to the characteristics of the tire (the larger the wheel slip angle, the easier it is to reach the sideslip limit), the inner wheel is more likely to side slip. Therefore, to increase the turning limit ability of the vehicle, it is necessary to appropriately reduce the differences between the inner and the outer wheel angle.

To pursue the ultimate capacity of the vehicle, some vehicles sometimes choose to use the anti-Ackerman corner (the characteristics of the tire, the outer wheel load is large, and the larger slip angle can provide greater lateral force).

Household vehicles are often used in low-speed turning and high-speed turning conditions, so the steering design is between Ackerman steering and parallel steering trapezoid.

2.2 Automatic Packing Technical Problems

The automatic parking process can be divided into three parts, detecting parking space, planning path, and tracking path.

(1) Detecting parking space is to use the body sensor (ultrasonic radar, camera) to detect the distance between the vehicle and the roadside vehicle to determine if the length of the parking space meets the need of parking.

(2) Planning path is that the on-board processor determines the current location of the vehicle, the location of the definte parking space, and the surrounding environmental parameters to plan the best parking path based on the detected distance, the relative position of the target parking space, and other information.

(3) Path tracking is to control the vehicle to execute path planning, convert related strategies into electrical signals to the actuator, and guide the car to park according to the planned path according to the instructions.

Technical Difficulties

(1) Detection and recognition accuracy of parking spaces

Ultrasonic radar has a large scattering angle and low directivity. When measuring a long-distance target, its echo signal will be relatively weak, which affects the accuracy. For temperature sensitivity, under different temperature conditions, the ultrasonic propagation speed is different, the measurement distance is also different. The large amount of data acquired by the camera and a large amount of image information require a large amount of computing power. And it is easy to be affected by the environment. The surface of the radar is stained, the water is attached to it, and the automatic parking fails when the camera is humid or wet.

(2) Path planning

Path planning is mainly realized through control algorithms. There are three steps in the process, namely the adjustment of the starting position outside the parking space, the parking entry, and the posture adjustment in the parking space.

a. The outside initial position the parking space should be set in the control algorithm for the distance, position, and other conditions so that the vehicle position satisfies the parking conditions;

b. Establish a model during the parking phase and make reasonable path planning;

c. In the adjustment stage, a systematic analysis should be carried out on the position and posture of the vehicle body related to the parking space, and a plan for vehicle adjustment in the parking space should be formulated to ensure that the vehicle satisfies the conditions.

The premise that these strategies can be realized is that a large amount of parking data is needed for analysis, and it must be combined with the specific parking conditions at the time.

2.3 The purpose of the mathematical model

The mathematical model of automatic parking is to solve the pain points of daily work and life. Its application areas are usually the ground or underground parking lots of office buildings or large shopping malls. It is necessary to realize the functions of finding and parking in the garage. To safely drive to the location where it is needed, it is necessary to improve the long-distance perception ability of the car, solve the positioning and map construction problems when moving in an unknown environment, plan a more reasonable path, and drive to an empty parking space.

3. Mathematical modeling of visual positioning

3.1 Basic Knowledge of visual positioning

The simplest method we currently use is to use five points, four points for calibration, and one point for testing, using the object method to make only these four control points appear in the scene.As shown in the figure, I selected four points on the calibration board, and at the same time determined its coordinate’s u, v in the image coordinate system, and coordinates in the world coordinate system X, Y, Z in the car. We can get R, T between the two coordinate systems

3.2 Schematic

The simplest method we currently use has five points, four points for calibration, and one point for testing, using the object method to make only these four control points appear in the scene. As shown in the figure, I selected four points on the calibration board, and at the same time determined his coordinate's u, v in the image coordinate system, and coordinates in the world coordinate system X, Y, Z in the car. We obtain R and T between the two coordinate systems. The schematic diagram is as follows 3.1:

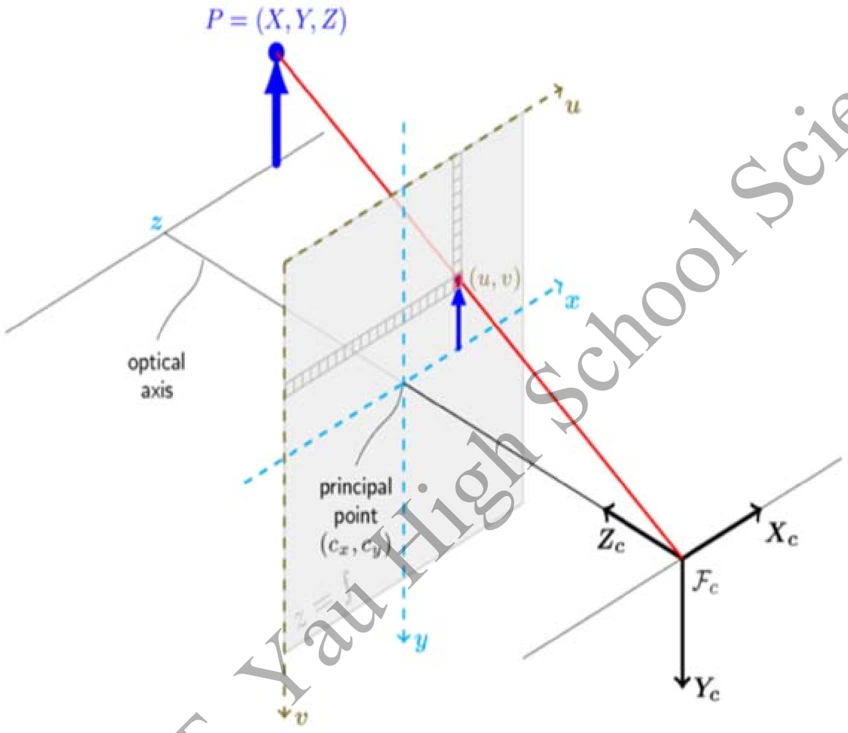


Figure3.1 Schematic diagram

3.3 Calculations

The following formula is the conversion formula between image coordinates and world coordinates. The first matrix is the camera internal parameter matrix, and the second matrix is the camera external parameter matrix. In this formula, the image's coordinates are known and the camera internal parameter matrix has been obtained through calibration. Lack of s.

$$s\begin{bmatrix}u\\v\\1\end{bmatrix}=\begin{bmatrix}f_x&0&c_x\\0&f_y&c_y\\0&0&1\end{bmatrix}\begin{bmatrix}r_{11}&r_{12}&r_{13}&t_1\\r_{21}&r_{22}&r_{23}&t_2\\r_{31}&r_{32}&r_{33}&t_3\end{bmatrix}\begin{bmatrix}X\\Y\\Z\\1\end{bmatrix}$$

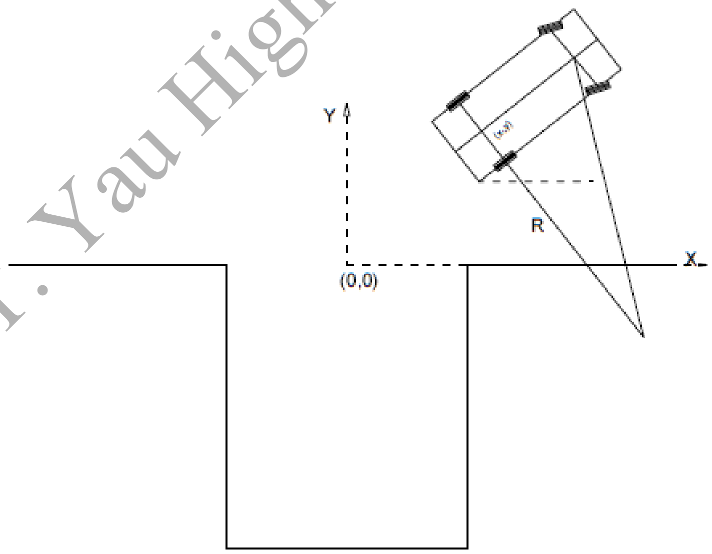
The conversion formula is converted to the following left formula, where M is the parameter matrix in the camera, R is the rotation matrix, t is the translation matrix, and Z is the distance between the origin of the actual coordinate system and the origin of the camera coordinate system on the Z-axis. After the left formula is transformed, the right formula can be obtained. In the right formula, when R, M, t, and Z are known, S is the only variable, so S can be obtained

$$s\begin{bmatrix}u\\v\\1\end{bmatrix}=M\left(R\begin{bmatrix}X\\Y\\Z_{\text{const}}\end{bmatrix}+t\right)$$

$$R^{-1}M^{-1}s\begin{bmatrix}u\\v\\1\end{bmatrix}=\begin{bmatrix}X\\Y\\Z_{\text{const}}\end{bmatrix}+R^{-1}t$$

4. Establishment of mathematical model of automatic parking

See figure 4.1



Figure

4.1 Automatic parking coordinate system establishment

4.2 Ackermann steering geometric model

Kinematic model based on Ackermann steering geometry According to Ackermann steering geometry, the Ackermann rotation angle is approximately equal to the front axle center rotation angle. The steering center O is on the extension line of the rear axle, and the steering points to the side, as shown in Figure 4.2

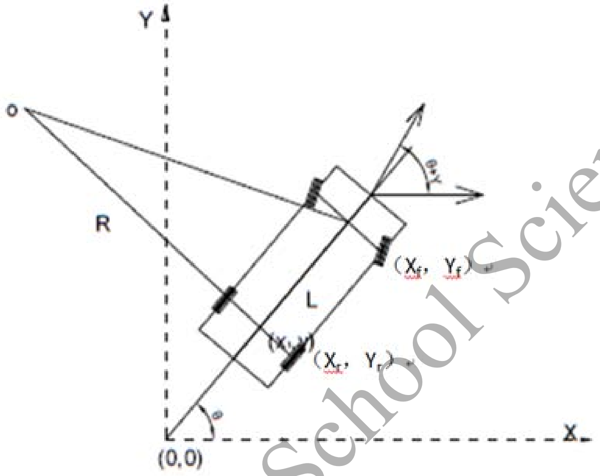


Figure 4.2

It can be seen from the figure that the turning radius R corresponding to the center of the rear axle is:

$$R=\frac{L}{\tan\varphi} \qquad \textbf{(4-1)}$$

In the formula: φ is the rotation angle of the center point of the front axle, and L is the wheelbase of the vehicle.

Car parking is a low-speed driving process. In low-speed driving on a flat road, the side slip of the wheels can be ignored. The wheels only roll and turn. The vertical speed of the rear axle center is 0. The equation is expressed as:

$$y_r' \cdot \cos\theta - x_r' \cdot \sin\theta = 0 \quad \textbf{(4-2)}$$

Where: θ is the heading angle of the vehicle.

According to the geometric relationship, the coordinate relationship between the center point of the front axle（x_{f}, y_f）and the center point of the rear axle（x_r, y_r）is as follows:

$$\begin{cases} x_r = x_{\mathrm{f}} - L \cdot \cos\theta \\ y_r = y_f - L \cdot \sin\theta \end{cases} \quad \textbf{(4-3)}$$

Derivation from both sides to get the speed relationship:

$$\begin{cases} x_r' = x_f' - \theta' \cdot L \cdot \cos\theta \\ y_r' = y_f' - \theta' \cdot L \cdot \sin\theta \end{cases} \qquad \textbf{(4-4)}$$

Substitute equation (4-4) into equation (4-2) to get

$$x_f' \cdot \sin\theta - y_f' \cdot \cos\theta + \theta' \cdot L = 0 \text{（}\textbf{4-5}\text{）}$$

From the vehicle coordinate system, the horizontal and vertical speeds of the front axle center are:

$$\begin{cases} x_f' = v_f \,.\, \cos(\theta + \varphi) \\ y_f' = v_f \cdot \sin(\theta + \varphi) \end{cases} \text{（}\textbf{4-6}\text{）}$$

Where: v_f is the speed of the center point of the front axle of the vehicle.

Incorporating formula (2-6) into formula (2-5) to obtain the angular velocity of the center point of the front axle of the car is:

$$\theta' = v_f \cdot \frac{\sin\theta}{L} \text{（}\textbf{4-7}\text{）}$$

Vehicle kinematic model

$$\begin{cases} x_r' = v_f \cdot \cos\theta \cdot \cos\varphi = v_r \cdot \cos\theta \\ y_r' = v_f \cdot \sin\theta \cdot \cos\varphi = v_r \cdot \sin\theta \\ \theta' = \sin\theta / L = v_r \cdot \tan\varphi / L \end{cases} \qquad \text{（}\textbf{4-8}\text{）}$$

In the formula, v_r is the speed of the center point of the rear axle of the vehicle, Integrate the above formula to get:

$$\begin{cases} x_r = \int_0^r v_f \cdot \cos\theta \cdot \cos\varphi \cdot dt \\ y_r = \int_0^r v_f \cdot \sin\theta \cdot \sin\varphi \cdot dt \\ \theta = v_r \cdot \frac{\tan\varphi}{L} \cdot \theta_0 \end{cases} \qquad \text{（}\textbf{4-9}\text{）}$$

Thus, the trajectory of the center of the rear axle is as follows:

$$(x_r - L \cdot \cot\varphi \cdot \sin\theta_0)^2 + (x_r - L \cdot \cot\varphi \cdot \cos\theta_0)^2 = (L \cdot \cot\varphi)^2 \qquad \text{（}\textbf{4 -10}\text{）}$$

When the vehicle speed is constant, the trajectory of the center of the rear axle（x_r,

y_r） is a circle, and the curvature of the trajectory circle is only related to φ and L, and has nothing to do with v. The derivation of the above formula and knowledge of dynamics shows that when the vehicle is moving at low speed, and when the steering wheel angle of the vehicle is constant, the track of the center point of the rear axle of the vehicle is a fixed circle.

4.3 Vehicle location grouping

When grouping the vehicle positions according to the distance between the vehicle and the parking space, the garage environment is first analyzed to determine the feasible starting area for parking, and then the parking edge analysis is performed to determine the collision boundary adjustment. Finally, because there are different optimal parking modes when the distance between the vehicle and the parking space is different, this paper classifies the parking mode and combines the edge analysis of the car to group the vehicle positions.

4.3.1 Classification of parking modes

The analysis shows that when the vehicle is in different positions, the path length and time required for different parking modes are different. This article divides the parking mode of "one forward and two retreats" into three; parking mode M1 requires to Determine the forward and backward corners. After passing through two arcs, the vehicle body and the parking space are perpendicular to the position, and when the vehicle is directly reversed and warehousing; in parking mode M2, after determining the front wheel corner, walk through a circular arc, and then the vehicle body It is perpendicular to the parking space and directly reverses into the garage; the parking mode M3 is similar to M1, except that the first section of the trajectory is a downward driving arc.

The schematic diagram of the three parking modes is as shown in 2.4. This article stipulates that the initial direction of the front of the car is all to the right:

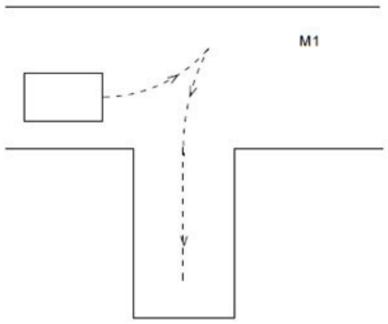


Figure 4.3.1 Route diagram of parking mode（M1)

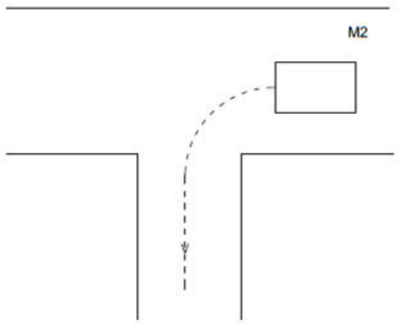


Figure 4.3.2 Route diagram of parking mode（M2）

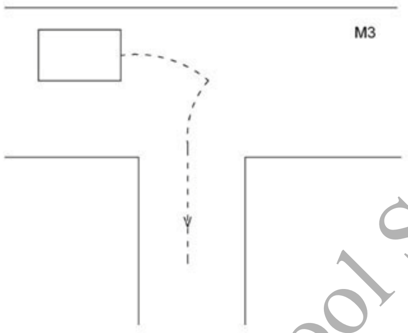


Figure 4.3.3.Route mode of parking mode(M3)

4.3.2 Parking start area

According to the experience of skilled drivers, parking the vehicle as much as possible on the axis of the parking space during the parking process, on the one hand, to leave as much space as possible for the vehicle to leave the parking space, on the other hand, to ensure that the vehicle has enough space for the car door. As shown in Figure 2.5, Figure a shows that the vehicle is in a critical collision state with the obstacle at point B, and Figure b shows that the vehicle is in a critical collision state with the edge of the road.

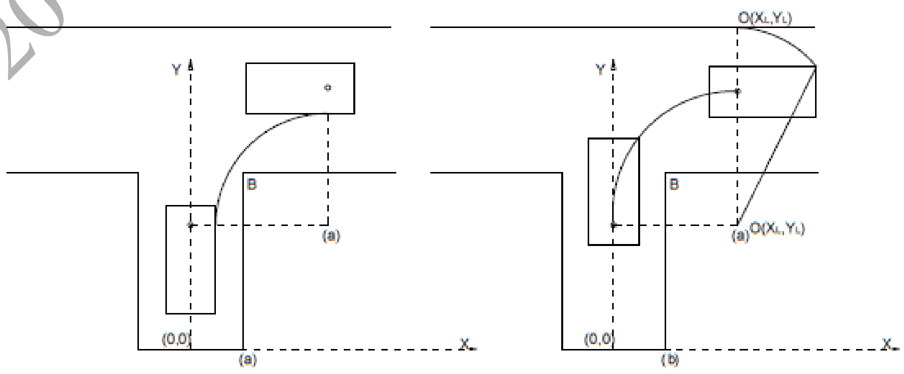


Figure 4.4 Limit position analysis

This article takes the parking space on the right side of the vehicle as an example of research. According to the research on the actual parking problem, the starting position of the vehicle parking is between the lower-left extreme position and the upper right extreme position.

4.3.3 Parking edge analysis

In the process of reversing, to ensure that the vehicle is parked on the central axis of the parking space as much as possible, and does not collide with boundary B, the critical state is shown in an in Figure 2.5, with:

$$(x-x_0)^2+(y-y_0)^2=\left(R_{\min}-\frac{L_3}{2}\right)^2 \qquad \textbf{(4-11)}$$

In the formula, the center of the car steering circle is（x_0, y_0）. L_3 is the width of the vehicle. $R_{\min}$ is the minimum turning radius of the vehicle.

For example, if it just touches the edge point B（W /2, H）, then:

$$\begin{cases} x_0=R_{min} \\ y_0=H-\sqrt{\left(R_{min}-\frac{L_3}{2}\right)^2-\left(\frac{W}{2}-x_0\right)^2} \end{cases} \qquad \textbf{(4-12)}$$

In the formula, H is the length of the parking space and W is the width of the parking space.

The minimum distance between the right side of the vehicle and the inside of the curb is: d_{1min}

$$d_{1min}=y_0+R_{\min}-H \qquad \textbf{（4-13）}$$

To ensure that the car does not collide with the curb and the car is parked on the central axis of the parking space, as shown in b in Figure 6.6, the area swept by the vehicle does not coincide with the curb. The turning center of the vehicle is （x_1, y_1）. The trajectory equation of point a of the body is:

$$(x-x_1)^2+(y-y_1)^2=(R_a)^2 \qquad \textbf{（4-14）}$$

In the formula, R_a is the turning radius at point a.

The critical coordinates of the vehicle contacting the curb are $(x_0, H+b)$. The center of rotation （x_1, y_1） is:

$$\begin{cases} x_1 = R_{min} \\ y_1 = H + b - R_a \end{cases} \quad \textbf{（4-15）}$$

After the above analysis, it can be obtained that the vehicle's reserve parking area is close to 0. The center of the vehicle's rear axle（x_r， y_r）should meet:

$$\begin{cases} x_r > R_{min} \\ y_0 + R_{min} < y_r < y_1 + R_{min} \end{cases} \quad \textbf{（4-16）}$$

The minimum distance d_{2min} between the left side of the vehicle and the curb is

$$d_{2min} = H + b - y_1 - R_{min} \qquad \textbf{（4-17）}$$

The width of the middle area is

$$d_3 = b - d_{1\text{min}} - d_{2\text{min}} = y_1 - y_0 \qquad \textbf{（4-18）}$$

Use Matlab to solve the model to get the vehicle position grouping situation, and the solution is $d_{1\text{min}}$=1173mm, $d_{2\text{min}}$=867mm, $d_{3\text{min}}$=5960mm In area 1 and area 3, due to edge conditions, only the parking mode of mode M1 can be used to reach the target parking space. In area 2, mode M1 and mode M2 can be used, but the optimal parking mode is M1, as shown in the intuitive diagram Figure 2.6,

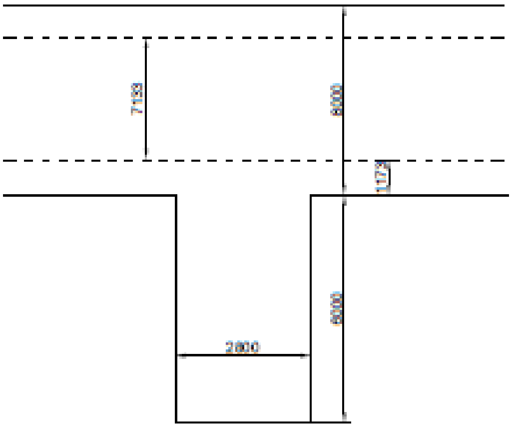


Figure 4.5 Group map of vehicle location

5. The target model is solved

The problem requires each group to find the best ideal starting point for parking. The selection of the ideal starting point for parking should satisfy that there is an optimal parking strategy at this point. Since the speed does not affect the selection of the ideal starting point, it is Ask, our home is a constant speed during the car parking process, and analyze which variables will affect the length of the parking path. Therefore, this question is transformed into an optimization problem with the shortest parking path as the goal.

5.1 Decision variables

According to the kinematics model established above, to determine the position of the vehicle at each moment in the reversing process, this article first determines the decision variable as the center coordinate of the rear axle [xr(φ,θ,v_f,t),yr(φ,θ,v_f ,t)) and the center of the rear axle correspond to the turning radius R(φ).

5.2 Determination of the target function

To enable rapid and variable vertical parking of the vehicle, the vehicle is required to be able to stop from the ideal starting point$[\mathrm{xr}\ (\varphi,\theta,\mathrm{v}_\mathrm{f},0),\mathrm{yr}(\varphi,\theta,\mathrm{v}_\mathrm{f},0)]$ Reach the target parking area with the shortest path length traveled by the vehicle known from the title. When the process of parking the car satisfies "one forward and two retreats" the path that the car travels includes: the sum of the three parts of the first arc S1, the second arc S2, and the vertical reverse S3.According to the kinematics model, the trajectory before parking is an arc, and the path length of each part is related to the center angle α of the starting point and the ending point, θ is the steering angle speed, then:

$$\mathrm{a} = \theta' \cdot \mathrm{t} \qquad （5\text{-}1）$$

The length of the arc is

$$\mathrm{S}_1 = \frac{\alpha_t}{360^\circ} \cdot 2\pi \cdot (L.\cot\varphi)(t = 1,2) \qquad （\mathbf{5\text{-}2}）$$

In the formula, L is the wheelbase of the vehicle, and φ is the rotation angle of the center point of the front axle.

The last segment of parking is vertical reverse storage, then

$$s_3 = v_f \cdot t_3 \qquad \textbf{（5-3）}$$

The goal of this model is the smallest sum of paths, so the objective function is:

$$\min S = \sum_{i=1}^{3} s_i \qquad \textbf{（5-4）}$$

In the formula, S represents the sum of the paths, that is, the center track distance of the rear axle of the car.

5.3 Analysis of constraints

(1) Vehicle **parameter constraints**

When the vehicle is turning, there is a minimum turning radius $\mathrm{R_{min}}$. At this time, the front wheel angle of the vehicle is the largest, then:

$$L \cdot \cot\varphi \geq R_{\min} \qquad \textbf{（5-5）}$$

(2) Vehicle collision constraints

During the parking process, the vehicle may collide with the boundary of the garage. Based on safety considerations, the collision situation of the vehicle during parking is analyzed, as shown in Figure 2.7.

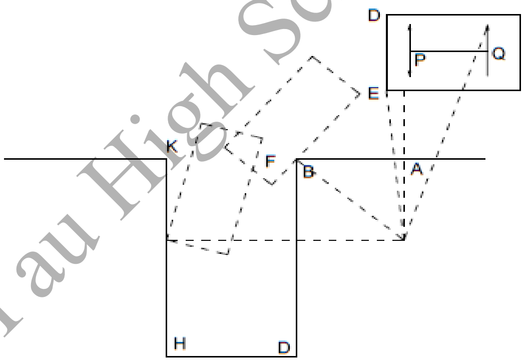


Figure 5.1 Schematic diagram of parking collision

There are two types of collision conditions

（1）Analysis of the collision between the vehicle and the right boundary of the parking space

Right edge collision

The motion of the vehicle is approximated as a circular motion without wheels slipping. The trajectory of the vehicle at point B is an arc with a radius of OB, and because the body is on the left side of point B, point B may collide with FG. As shown in Figure 5.2

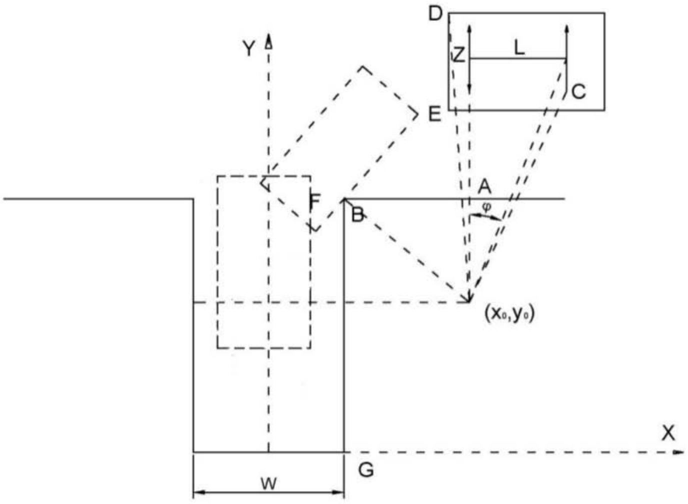


Figure 5.2

Knowing that L is Axle, L_f is the front width, w is the width of the parking space

First, based on the Ackermann steering principle, in the right triangle OZQ, there are:

$$R = OZ = L.\cot\varphi \qquad (5\text{-}6)$$

Secondly, from the vehicle geometry relationship, the formula can be derived;

$$OA^2 + OB'^2 = 0B^2 \qquad (5\text{-}7)$$

$$OZ = OB + \frac{1}{2}L_f \qquad (5\text{-}8)$$

$$OB = OB' = OZ - \frac{1}{2}L_f \qquad (5\text{-}9)$$

$$AB' = X_0 - \frac{1}{2}w \qquad (5\text{-}10)$$

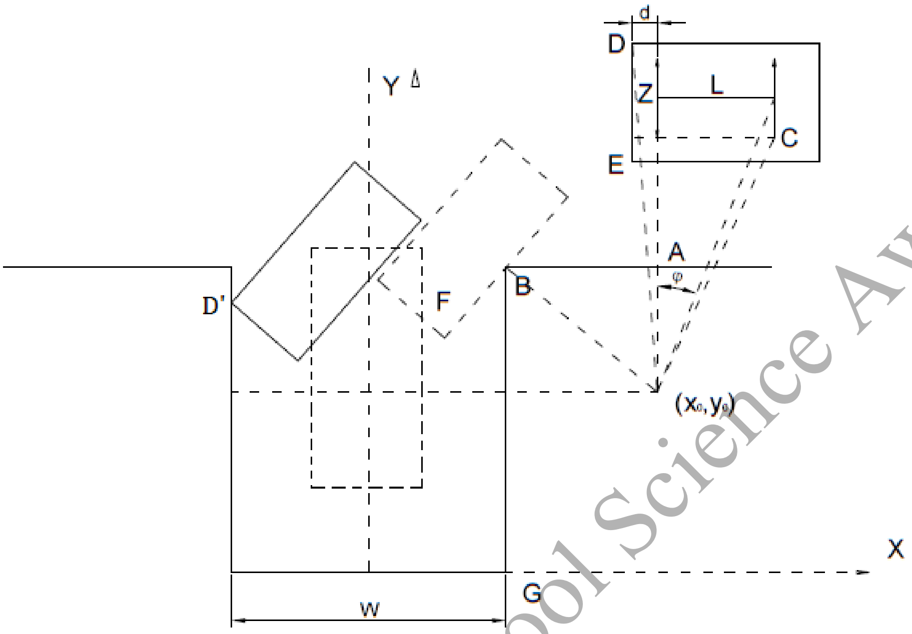
$$0A = 0Z - ZA = 0Z - y_0 \qquad (5\text{-}12)$$

$$\left(x_0 - \frac{1}{2}w\right)^2 + (R - y_0)^2 \le \left(R - \frac{1}{2}L_f\right)^2 \qquad (5\text{-}13)$$

Finally, the constraint conditions that the vehicle does not collide with the left boundary of the target parking space are:

$$\left(x_0 - \frac{1}{2}w\right)^2 + (R - y_0)^2 \le \left(R - \frac{1}{2}L_f\right)^2 \qquad (5\text{-}14)$$

(2) The collision analysis between the vehicle and the left boundary of the parking space, as shown in Figure 5.3



As shown 5.3

First, analyze the critical condition of the collision between the vehicle and the left boundary of the target parking space. As shown in Figure 2.9, the left rear corner point D of the car is located at the leftmost position of the vehicle body, and the critical condition for the car to enter the parking space just past the KH satisfies the equation:

$$OD' = 0D = \sqrt{d^2 + \left(R + \frac{1}{2}L_f\right)^2} \qquad （\mathbf{5\text{-}15}）$$

Then, analyze the critical conditions for the collision between the vehicle and the left side of the parking space：

$$\mathrm{x}_0 + \left(\frac{1}{2}w\right) \geqslant \sqrt{\left(R + \frac{1}{2}Lf\right)^2 + d^2} \qquad （\mathbf{5\text{-}16}）$$

Finally, the constraint conditions that the vehicle does not collide with the left boundary of the target parking space are:

$$x_0 + \left(\frac{1}{2}w\right) \geqslant \sqrt{\left(R + \frac{1}{2}Lf\right)^2 + d^2} \qquad \text{(5-17)}$$

(3) Safety boundary constraints of vehicle parking spaces

Considering the safety of parking and the convenience of drivers getting in and out of the car, a certain safe distance should be reserved between vehicles when parking. Therefore, the vehicle in the parking space should meet the following 2.10 boundary conditions:

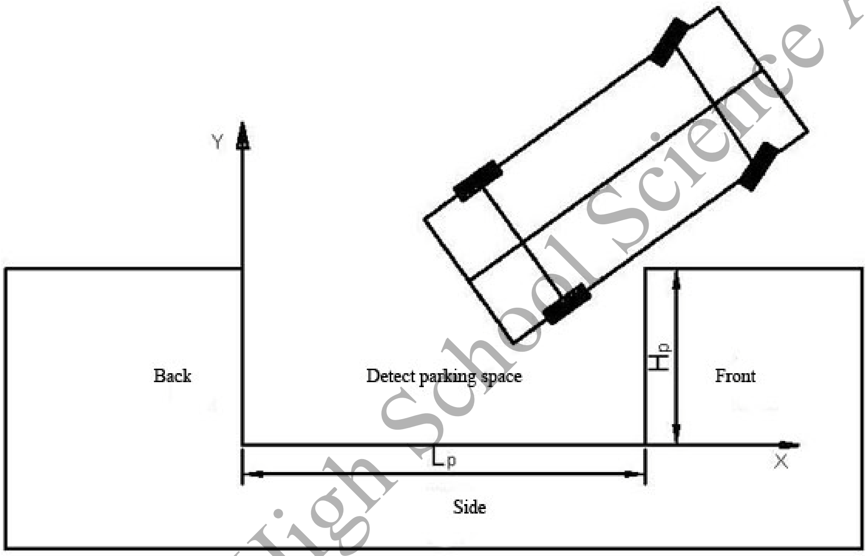


Figure 5.4 Schematic diagram of parking space and vehicle pose coordinates

For a valid initial position, several conditions need to be determined (2.11 below):

1) After starting to park, fill the steering wheel, and the vehicle should avoid colliding with other vehicles in the moving parking space during reversing.

2) During parking and warehousing, the vehicle cannot collide with the trousers or the bottom of the warehouse.

3) The minimum turning radius is used to ensure the shortest trajectory, and also to restrict the size of the parking space

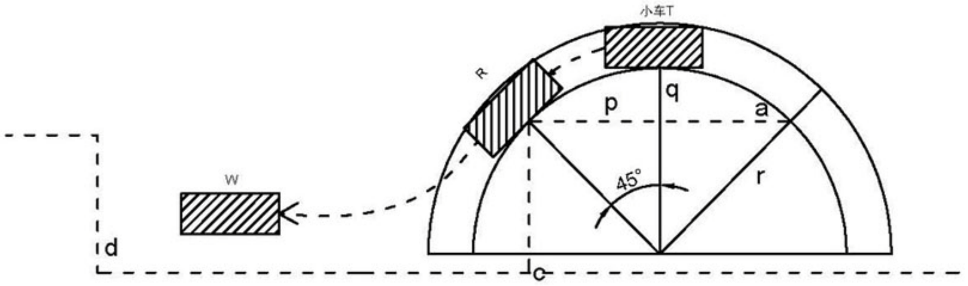


Figure5.5 Determination of the minimum parking space

6. Application of reinforcement learning in autonomous parking technology modeling

In the verification of the above mathematical models, it is necessary to test the driverless parking technology constantly under all kinds of conditions of driving, so as to guarantee its security level can be higher than the operation of human drivers. However, there are many situations that we can't encounter. Such as heavy rain, heavy snow, and complex traffic environment. So we need simulation to fill in the validation. The virtual road test using unmanned driving simulator can effectively test dangerous or uncommon driving scenes. The flexibility and versatility of virtual road test play an important role in the development of automatic driving technology, and it is one of the important means to accumulate the test mileage of unmanned vehicles.

At present, the environment of driverless parking is mainly the underground garage with small space and dim light. There have been reports of automatic parking cars accidentally running over young children and killing them due to blind spots. Automatic parking technology mainly relies on two detection systems which play an important role respectively in software and hardware: one is millimeter wave radar based on perception system, and the other is mainly based on computer vision algorithm. Compared with the computer vision perception system, the use of MMW radar undoubtedly increases the hardware cost. Meanwhile, MMW radar has a certain error in the distance measurement of near objects, which has a great impact. We propose an optimized automatic parking strategy based on reinforcement learning, which greatly improves the efficiency and safety of automatic parking, and implement the reinforcement learning parking model with Python.

I realized the automatic parking trajectory planning strategy based on deep reinforcement learning, which mainly includes the following steps:

1. Establish a two-dimensional virtual parking environment based on the information obtained from the visual system;

2. Simulate vehicle parking movement, obtain steering wheel angles, and update two-dimensional virtual parking environment;

3. Establish an automatic parking model based on deep reinforcement learning, adopt the method of deep reinforcement learning, obtain some corresponding steering wheel angles in the process of automatic parking, and generate automatic parking track;

4. Judge whether there is a collision in the virtual parking environment. If there is a collision, initialize the two-dimensional virtual parking environment and start the next parking training; otherwise, proceed to the next step;

5. Plan the automatic parking track, and select the optimal automatic parking track by comprehensively considering the length and time of the track.

This strategy uses deep reinforcement learning model to train tens of thousands of automatic parking processes in the virtual parking environment. The whole training process basically covers the corresponding steering wheel Angle operation under different positions of the vehicle in the virtual parking environment, and the experiment is universal.

Here is a picture of a python driverless car simulator running on your own computer.

For specific video, please refer to the uploaded video MP4 file:
driverless 1, automatic parking 2

Figure 6.1driverless

Figure 6.2 Automatic parking into parking lot

Figure 6.3 Looking for empty parking spaces

Figure 6.4 Avoid opposite vehicles

7. Prospects and future of driverless parking technology

If self-driving cars are to be commercialized, automatic parking has always been considered a good entry point. Compared with the self-driving automobile, the use of automatic parking is relatively simple, and the technical difficulty is relatively small. However, automatic parking, especially in large cities, where there are many cars but limited parking space, encounters some more complex scenarios.

For example; oblique parking spaces, narrow parking spaces, or no reference objects for positioning around the parking spaces. Parking is not only a test of driving skills and it may even cause congestion or traffic accidents.

How to solve the actual pain points of users in-car use is a technical problem that needs to overcome for automatic parking. Of course, to achieve this, the technical requirements are also higher.

At present, there are two ways to achieve this. The first one is to improve the environment perception and computing power of the vehicle itself, to make the vehicle smart enough to cope with the scene of mixed traffic in the parking lot, and to decide on the avoidance route and the safety of passersby.

Autonomous parking; the other is through the combination with the parking lot, that is, the car is loaded with sensors, and the parking lot section assists, especially when the parking lot signal is weak if the parking lot and the vehicle can achieve information interaction, Will greatly simplify the difficulty of parking.

However, although the above two methods are the more mainstream solutions for smart parking at present, the challenges they face cannot be ignored. For example, taking the car terminal as the leading core will increase the cost of producing bicycles by the OEM, while taking the parking lot as the center will face the problems of inconsistent agreements, inconsistent standards, and high renovation costs at this stage. From this point of view, the majority of users who want to be "hands-off drivers" when parking, still need to wait for a while.

What is gratifying is that automatic parking has been launched in the last two years with corresponding mass-produced models. Although the practicality and parking accuracy still need to be improved, the mass production can be carried out, indicating that it has taken a big step in the driverless link. With the efforts of many manufacturers, automatic parking technology is moving towards more intelligent, that is, fully autonomous parking.

Looking forward to the day when automatic parking is implemented.

Acknowledgement

I have been fascinated by formula racing since childhood, and later developed a strong interest in autonomous driving. It's hard to imagine how the world would change if autonomous driving technology could replace the traditional driver. My interest led me to self-study and complete the knowledge of mathematics, physics and programming in the field of automatic driving.

I would like to thank Teacher Zhang Hongwei for guiding me to explore this field and giving me free help. In the first year of high school, I taught myself Python programming and learned to write code according to requirements. Under the leadership of Mr. Zhang, I completed the deep learning and intensive learning in two months, completed the mathematical modeling of automatic parking, and implemented the user-friendly intensive learning parking model with Python. Mr. Hu often helps me revise my paper late at night and patiently guides me to carry out experiments and finish the paper according to the strict academic requirements. His erudition and preciseness set an example for me, and his academic training will benefit me all my life.

Finally, I would like to thank The QiuChengtong High School Science Award, which has enabled me to launch an exciting exploration in my favorite field.In middle school, my greatest pleasures were mathematics and programming. Matlab, Python, one programming language after another became an extension of my thinking and a tool to solve problems. As I learn more and more programming languages, I realize that algorithms are the essence of all programs, the bridge between abstraction and reality. We will use the algorithm model to summarize the reality of the problem, abstract, programming computing force can be applied form, mathematics is the underlying law of the algorithm. This subject study made me further think about how to use mathematical laws to improve the programming algorithm, and then solve practical problems, and I gained a lot.

Once again, I would like to express my gratitude to the teachers and family members who helped me in my research.

Team member introduction

Zimeng Yi, female, is a senior grade three student in Shenzhen senior high school. She has a wide range of hobbies and strong logical thinking. She especially loves number theory and physics science. She is the president of the school's physics bridge model society.

Upgraded from AMC 12 (American Math Competition) to AIME（American Mathematics Competition) in 2018;

First place in Shenzhen bridge model competition in 2019;

Won the first prize and the second place in the National Youth building model education competition of "building a home together" and in 2019;

Ranked the top 100 in the individual competition area of the ARML(American Regions Math League）in 2020.

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