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Title: A Tripartite Game Analysis on Evolutionary Equilibrium of Carbon Emission

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Abstract: Governments implement various policy tools to achieve carbon emission reduction objectives. These policy objectives can only be achieved through impacting the producer and retailer, two key players of the low-carbon supply chain. This paper studies the government-leading tripartite game approach among the government, the producer, and the retailer. Firstly, the Stackelberg game model is applied to reach the equilibrium point among three parties. Then the Evolutionary game theory is used to achieve the evolutionary stable strategy. Finally, a stimulation study is conducted to demonstrate the tripartite evolutionary equilibrium model. The results show that each of government's carbon emission strategy generates its corresponding equilibrium point, and the strict carbon emission strategy will lead the producer to adopt advanced technologies to reduce carbon emission. In addition, the stability of the equilibrium point among the government, producers and retailers will not be affected by whether the government implements tolerant or strict policy. The consumer's sensitivity coefficient to carbon emission and the consumer's sensitivity coefficient to marketing effort affect the stability of the equilibrium point of the system evolution.

Keywords: Carbon Emission, Low-carbon Supply Chain, Tripartite Game, Stackelberg Game, Evolutionary Stable Strategy

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1. Introduction

In the face of global warming and environmental degradation, approaches for reducing the amount of carbon emissions caused by traditional energy consumption has become a global objective. To reduce carbon emissions is not only to protect the homeland on which human live, but also to guarantee the sustainable development of social economy. Many countries have started taking actions. Some European countries such as Germany has promised to ban gasoline and diesel cars from 2030 and reduce its carbon footprint by 80 to 95 percent by 2050. China officially initiated the pilot program of carbon emission trading in 2011, and launched a nationwide carbon trading market with an open trading system in July 2021. In September 2020, Chinese formally proposed to target reaching a carbon peak by 2030 and a carbon neutrality by 2060, A.K.A the "30-60 Target".

Governments around the world have been trying to cut carbon emissions of high-polluting industries through implementing carbon policies. Building the low-carbon supply chain has increasingly become the key and effective element to achieve government's policy objectives, especially when it combines with economic incentives. A low-carbon supply chain can not only reduce the damage to the environment, but also optimize the resource allocation, eventually achieving a synchronous development of producers and the protection of environment .

Achieving low-carbon supple chain starts with the government implementing carbon emissions abatement policies, which includes: levy carbon tax, set carbon quota, carry out carbon trade, implement green electricity premium, and others. These policies can only achieve their goals by effectively influencing the producers and retailers (or distributors) of products. At the same time, education and publicity should be strengthened to cultivate consumers' awareness of environmental conservation, so as to affect the supply side from the demand perspective. Typically, government's carbon emission targets do not directly affect individual consumers, but rather do so indirectly through the distribution of products (retailers), such as providing green subsidies or coupons.

Due to competing interests among enterprises, they would react differently toward the government's policies, resulting in a game between the government and the producer, the producer and the retailer, as well as the retailer and the government. A tripartite game model will be important to understand how the government's carbon emission policies will effect the producer and the retailer, and eventually achieve its policy objective.

In this paper, the Stackelberg game model is mainly used to analyze the game approach among the three parties, of which the government is in the leading position. However, in the real world, due to reasons such as lack of information transparency or inefficiency of policy communication, it is difficult for the three parties to obtain the optimal strategy or optimal equilibrium directly. They will constantly adjust their own strategies based on vested interests and make decisions dynamically. As such, the Evolutionary game theory is also applied to further analyze the evolutionary stability strategy. In the final part of this paper, the Matlab tool is applied to stimulate the theory and analysis, demonstrating what would be in the real world.

Existing studies mainly focus on the game analysis between two parties, either between the government and the producer, or between upstream and downstream supply chain. Wang and Cheng (2019) conducted game studies on the producer's emission reduction and the retailer's low-carbon marketing strategies. In their study, Stackelberg game model and evolutionary stability strategy are applied. However, there is a lack of analysis and theoretical model that includes government, producer and retailer to a tripartite game system. This paper leveraged existing researches in this field, including the model approach, and certain key assumptions and conclusions conducted by Wang and Cheng (2019), but expanded to a government leading tripartite game study. The game strategy matrix expand from four to eight scenarios, and the stimulation also covers more complicated but closer-to-reality tripartite dynamic evolution. The focus on this paper is to bring the government into this game analysis and to understand its real world implication, rather than pure mathematical oriented model construction.

2. Literature Review

The main policies implemented by the govern to reduce carbon emissions mainly involve levying carbon tax, setting carbon quota, carrying out carbon trading system, implementing green electricity premium and other measures. Among them, carbon tax was first implemented in Northern European countries in the early 1990s, and has now been implemented in most developed countries and a few developing countries. Ye et al. (2022) found that implementing carbon tax separates it from other resource taxes like consumption tax, which would make it easier for the tax authorities to verify the credibility of the amount of the carbon emissions reported by enterprises themselves. However, it is still difficult to completely avoid under-reporting, making the policy less effective. Zhang et al. (2022) studied that, to efficaciously promote carbon emission reduction, the government tries to carry out the carbon trading system as another method, which facilitates the development green finance. In addition, Jiang and Sun (2016) researched that the government may set carbon quota and issue carbon credits according to the amount of the carbon emission submitted by producers. However, due to information gaps, it is difficult for the government to verify the real amount of carbon emissions, and therefore providing more credits than actually needed. Due to such uncertainty, the allocation of carbon credits would eventually be decided by games between the government and the enterprises. In this paper, we'd focus on finding the optimum solution in the government-enterprise game using the game theory, and then evaluate the stability of the equilibrium point using the the evolutionary stability strategy.

The government's influence over the low-carbon supply chain have been thoroughly studied. Most researches indicate that the government needs to carry out differential pricing mechanism according to the enterprise scale, in order to maximize economical efficiency. Liao et al. (2021) discussed the emission reduction decision of energy saving service companies and their upstream and downstream enterprises under the carbon trading policy. The research shows that the emission reduction input on the supply chain has an inverse relationship with energy saving efficiency. Zhang et al. (2022) used the projection contraction algorithm to solve the mathematical model of closed-loop supply chain satisfying Nash equilibrium, and concluded that in the environment of multiple demand markets and one carbon trading market, the government should set corresponding carbon allowances according to the business scope of enterprises. Yuan et al. (2022) constructed the pricing and decision-making model of the recovery supply chain by combining the carbon allowances allocated to enterprises by the government. By comparing the corresponding pricing changes of enterprises under different carbon allowances, they determined the appropriate amount of carbon allowances that the government should offer enterprises. Li et al.(2020) established a three-level closed-loop supply chain composed of the government, automobile producers and automobile recyclers, and discussed the incentive mechanism of manufacturing enterprises to their downstream recyclers under the mechanism of carbon trading market.

Based on the existing policies, a large amount of literatures have studied the decision-making process and result in supply chain. Xia et al. (2013) innovatively studied the pricing game between producers and retailers based on the cross-shareholding strategy, and verified the pricing rule of retailers by analyzing the changing trend of the cross-shareholding ratio on the profit value of producers and retailers under different carbon trading prices. Geels et al. (2017) put forward the "socio-technical" framework theory, hoping to constrain social groups to accelerate low-carbon transformation through mutual collaboration and influence. This theory breaks away from the traditional policy-oriented research, which is prospective and innovative in the field of low-carbon policy research. Diao et al. (2021) studied the pricing game process between producers and retailers for products with strong substitutions in a competitive environment, and proposed the use of payoff sharing contract to reasonably distribute the profits of both parties.

Previous studies focused mainly on the game between the government and the producer, the game between the producer and the retailer, or the game between the retailer and the government, respectively. Wang and Cheng (2019) conducted game studies on the producer’s emission reduction and the retailer’s low-carbon marketing strategies.

The innovation of this paper lies in including the government, the producer and the retailer to create a three party game, which helps to more clearly discuss the game in the entire supply chain. Furthermore, most precious studies mostly aimed at finding the equilibrium point of the game. However, in practice, producers and retailers may perform with bounded rationality and thus need trial and error before deciding on whether or not to reduce emissions and market in low-carbon ways. Therefore, the theoretical equilibrium might not be easily achieved. In this paper, we use the evolution stable strategy to evaluate the stability of the equilibrium point, which has great practical significance.

3. Evolutionary Game Model Description and Assumptions

3.1 Model introduction

Existing research mainly focuses on the coordination and decision-making of the supply chain under the circumstance of the carbon cap-and-trade policy, and the game of emission reduction between enterprises and the government, and between upstream and downstream enterprises. There is a lack of theoretical model that includes government, producer and retailer to a game system. Due to the fact that the central government and the local government are responsible for the management of the carbon market, and the government dominates the carbon market, it is necessary to regard governments as the main body of the game. Meanwhile, supply chains dominated by producers and retailers are of typical significance. This article takes the tripartite chain consisting of the government plus a single producer and a single retailer as the research object, so as to study the game between upgrading technology ("new technology") by the producer and adopting low-carbon marketing by the retailer under the government's different carbon policy approaches.

A tripartite system consisting of a government G, a producer S and a retailer R is the studying objective. The decision makers in this system are all bounded rational, and it is difficult to make an optimal choice in one decision-making. To obtain a stable equilibrium, it is necessary to adjust the strategy constantly. In addition to the original operation decision, the strategy set of government is $S_1 = \{X_1, X_2\}$. X_1 represents " Tolerant policy", which indicates a higher carbon limit （E_1）allocated by the government to producers, X_2 represents "Strict policy", indicating a lower carbon limit （E_2， $E_2 < E_1$） allocated by the government to producers; the strategy set of producers is $S_2 = \{Y_1, Y_2\}$. Y_1 represents adopting traditional technology, and Y_2 represents adopting new technology; the unit carbon emission of traditional technology is e_1^2, and the unit emission of new technology is e_2^2（$e_2^2 < e_1^2$）. There is a cost to upgrade technology. The strategy set of retailers is $S_3 = \{Z_1, Z_2\}$. Z_1 represents low-carbon marketing, and Z_2 represents non-low carbon marketing; non-low carbon emissions is e_1^3, and low carbon emissions is e_2^3（$e_2^3 < e_1^3$）.

3.2 Hypothesis and notation

Hypothesis 1 Suppose that a simple linear relationship exists between the demand function and the retail price of the product, and the producer's emission reduction efforts and the retailer's marketing efforts, that is, $D = a - bp + \lambda(e_1^2 - e_2^2) + \mu g$, where, a represents the market size of the product, b represents consumer price sensitive coefficient , λ represents consumer carbon emission reduction sensitive coefficient, and μ represents consumer marketing effort sensitive coefficient.

Hypothesis 2 When the producer adopts the traditional technology, the carbon emission per unit product is e_1^2. When the producer upgrades technology, the carbon emission per unit product is e_2^2, and $\left(e_1^2 - e_2^2\right)$ represents the unit product reduction in carbon emissions.

Hypothesis 3 When the producer upgrades technology, the carbon emission per unit product is reduced to $\left(e_1^2 - e_2^2\right)$, and the cost of emission reduction effort is $\frac{1}{2}k_1 e_1^2$. Producer profit = production income - cost of updating technology – cost of purchasing carbon (if technology is upgraded, then there is a cost; if technology is not upgraded, then there is no cost), that is, $u_2(x, y, z) = u_2^R(x, y, z) - u_2^T(x, y, z) - u_2^c(x, y, z)$, where the production income is $u_2^R(x, y, z) = (\omega - c)q$, and the cost of purchasing carbon is $u_2^c(x, y, z) = h\left[\left(e_1^1 - e_2^1\right)q - E_1\right]$ or $h\left[\left(e_1^1 - e_2^1\right)q - E_2\right]$.

Hypothesis 4 When the retailer adopts low-carbon marketing, the carbon emissions per unit product will be reduced to $\left(e_1^3 - e_2^3\right)$. g represents the retailer's marketing effort, and the marketing effort cost is $\frac{1}{2}k_2 g^2$, Retailer's payoff = Retailer's income - cost of low-carbon, i.e. $u_3(x, y, z) = u_3^R(x, y, z) - u_3^C(x, y, z)$, where the retailer income is $u_3^R(x, y, z) = (p - \omega)q$

Hypothesis 5 When the government chooses X_1 " tolerant policy", the carbon limit （E_1） allocated by the government to producers is higher, and when X_2 "strict policy" is selected, the limit （E_2， $E_2 < E_1$） allocated by the government to the producer is lower. The policy cost the government needs to pay is $u_1^C(x, y, z) = c_1$, and the government's payoff = social benefit - carbon emission - policy cost, that is, $u_1(x, y, z) = u_1^S(x, y, z) - u_1^{CE}(x, y, z) - u_1^C(x, y, z)$

Hypothesis 6 Suppose that social benefits depend on the overall reduction in carbon emissions, that is, $u_1^S(x, y, z) = \theta(T - T(y, z))$, where θ represents the government's payoff constant, and $T\left(Y_i, Z_j\right)$ is Carbon emissions of producers and retailers $T\left(Y_i, Z_j\right) = e_i^2 \cdot q + e_j^3$

Hypothesis 7 Define carbon emissions: $u_1^{CE}(x, y, z) = T\left(Y_i, Z_j\right) = e_i^2 \cdot q + e_j^3$

Table 1. Meaning of Parameters

Parameters	Parameter Meaning
θ	Government payoff constant
T	Total amount of carbon allowance currently available in the consumer carbon trading market
$T(Y_i, Z_j)$	Carbon emissions
ω	Wholesale price per unit
p	Retail price per unit
q	Product order（suppose $q = D$），$q > \omega$
D	Demand
e_1^2	Carbon emissions per unit when adopting traditional technology
e_2^2	Carbon emissions per unit when adopting new technology
e_1^3	Carbon emissions per unit when low-carbon marketing
e_2^3	Carbon emissions per unit when adopting non-low-carbon marketing
k_1, k_2	Cost constant
E_1	Carbon allowances when adopting “tolerant policy”
E_2	Carbon allowances when adopting "strict policy"
h	The trading price of carbon emissions per unit
g	Retailer's marketing effort
a	Market size of the product
b	Consumer’s price sensitive coefficient
λ	Consumer’s carbon emission reduction sensitive coefficient
μ	Consumer’s marketing effort sensitive coefficient

3.3 Benefit matrix of game subject's strategy

From the behavioral strategy sets of the government, the producer and the retailer, eight behavioral strategies for evolutionary games could be derived (as shown in Table 2).

With the model parameters shown in Table 1, the payoff to the government, the producer and the retailer under these eight behavioral strategies could be obtained. See Table 2 below for details.

Table 2. Payoff matrix

Behavioral Strategy	Government Payoff	Producer Payoff	Retailer Payoff
(X_1, Y_1, Z_1)	$\theta(T - e_1^2 \cdot q - e_1^3) - (e_1^2 \cdot q - e_1^3) - c_1$	$(\omega - c)q - \frac{1}{2}k_1e_1^2 - h[(e_0 - e_1)q - E_1]$	$(p - \omega)q - \frac{1}{2}k_2g^2$
(X_1, Y_1, Z_2)	$\theta(T - e_1^2 \cdot q - e_2^3) - (e_1^2 \cdot q - e_2^3) - c_1$	$(\omega - c)q - \frac{1}{2}k_1e_1^2 - h[(e_0 - e_1)q - E_1]$	$(p - \omega)q$
(X_1, Y_2, Z_1)	$\theta(T - e_2^2 \cdot q - e_1^3) - (e_2^2 \cdot q - e_1^3) - c_1$	$(\omega - c)q - h(e_1^2q - E_1)$	$(p - \omega)q - \frac{1}{2}k_2g^2$
(X_1, Y_2, Z_2)	$\theta(T - e_2^2 \cdot q - e_2^3) - (e_2^2 \cdot q - e_2^3) - c_1$	$(\omega - c)q - h(e_1^2q - E_1)$	$(p - \omega)q$
(X_2, Y_1, Z_1)	$\theta(T - e_1^2 \cdot q - e_1^3) - (e_1^2 \cdot q - e_1^3) - c_1$	$(\omega - c)q - \frac{1}{2}k_1e_1^2 - h[(e_1^2 - e_2^2)q - E_2]$	$(p - \omega)q - \frac{1}{2}k_2g^2$
(X_2, Y_1, Z_2)	$\theta(T - e_1^2 \cdot q - e_2^3) - (e_1^2 \cdot q - e_2^3) - c_1$	$(\omega - c)q - \frac{1}{2}k_1e_1^2 - h[(e_1^2 - e_2^2)q - E_2]$	$(p - \omega)q$
(X_2, Y_2, Z_1)	$\theta(T - e_2^2 \cdot q - e_1^3) - (e_2^2 \cdot q - e_1^3) - c_1$	$(\omega - c)q - h(e_1^2q - E_2)$	$(p - \omega)q - \frac{1}{2}k_2g^2$
(X_2, Y_2, Z_1)	$\theta(T - e_2^2 \cdot q - e_2^3) - (e_2^2 \cdot q - e_2^3) - c_1$	$(\omega - c)q - h(e_1^2q - E_2)$	$(p - \omega)q$

3.4 Construction of evolutionary game model

To achieve sustainable development and carbon emission reduction, the government released a set of carbon emission reduction policies, such as carbon cap-and-trade policy, carbon tax, etc. This article focuses on the decision-making of various players under the carbon cap-and-trade policy. The government gives producers a carbon emission quota, only when producers keep their carbon emissions lower than this quota, they may sell the remaining carbon emissions in the carbon market. On the other hand, if the producer's carbon emission exceeds this quota, the producer has the option to purchase carbon emission in the market. At the same time, since the government needs to consider the impact of the carbon cap-and-trade policy on the social and economic benefits as a whole, it will choose between tolerant and tight policies according to actual economic conditions. Based on the government-led Stackelberg game, we first determine the equilibrium point between producers and retailers under various government strategies, followed by finding the optimal government strategy. Lastly, we will analyze the stability of the equilibrium point according to the evolutionary game model.

In this case, firstly, we need to analyze what is the optimal strategy among the government, producers, and retailers, which involves the concept of equilibrium point. We'll dedicate a chapter to finding equilibrium.

However, in fact, due to factors such as information differences, it is impossible for the government, producers, and retailers to directly obtain the optimal strategy or optimal equilibrium. They constantly adjust their strategies based on vested interests and make decisions dynamically. Whether the system can reach equilibrium depends on whether the equilibrium point is stable. Therefore, we will dedicate a chapter to stability analysis.

4. Evolutionary Game Model Construction and Stability Analysis

Under the above model assumptions, we establish an evolutionary game model to further analyze the problem. First, we need to analyze the optimal strategy between the government, the producer, and the retailer, which involves finding the equilibrium point of the game. Next, we construct the evolutionary game model.

4.1 The construction of the evolutionary model

According to the Wang and Cheng (2019), in accordance with the existing carbon emission policies, once the producer has determined the the wholesale price per unit and the emissions of the new technology, and once the retailer has determined the retail price, the income matrix between players is decided, as shown in the following table (see Appendix for detailed formula):

Table 3. Combination of behavior strategies

Combination of Behavior Strategies	Government payoffs	Producer payoffs	Retailer payoffs
(X_1, Y_1, Z_1)	Γ_{11}^{g}	$\Gamma_{11}^{s} + hE_1$	Γ_{11}^{r}
(X_1, Y_1, Z_2)	Γ_{12}^{g}	$\Gamma_{12}^{s} + hE_1$	Γ_{12}^{r}
(X_1, Y_2, Z_1)	Γ_{12}^{g}	$\Gamma_{21}^{s} + hE_1$	Γ_{21}^{r}
(X_1, Y_2, Z_2)	Γ_{22}^{g}	$\Gamma_{22}^{s} + hE_1$	Γ_{22}^{r}
(X_2, Y_1, Z_1)	Γ_{11}^{g}	$\Gamma_{11}^{s} + hE_2$	Γ_{11}^{r}
(X_2, Y_1, Z_2)	Γ_{12}^{g}	$\Gamma_{12}^{s} + hE_2$	Γ_{12}^{r}
(X_2, Y_2, Z_1)	Γ_{12}^{g}	$\Gamma_{21}^{s} + hE_2$	Γ_{21}^{r}
(X_2, Y_2, Z_2)	Γ_{22}^{g}	$\Gamma_{22}^{s} + hE_2$	Γ_{22}^{r}

Based on the above, we will construct the evolutionary game model. We try to construct a complex dynamic equation of three parties in this paper. We assume that the probability of the government adopting the tolerant strategy X_1 is $x(0 \le x \le 1)$, the probability of the producer adopting the new technology Y_1 is $y(0 \le y \le 1)$, and the probability of the retailer adopting the low-carbon marketing strategy Z_1 is $z(0 \le z \le 1)$.

For the government, the expected payoff from the tolerant strategy is:

$$\bar{U}_1^g = yz\Gamma_{11}^g + y(1-z)\Gamma_{12}^g + (1-y)z\Gamma_{21}^g + (1-y)(1-z)\Gamma_{11}^g$$

The expected payoff of adopting a strict strategy are:

$$\bar{U}_2^g = yz\Gamma_{11}^g + y(1-z)\Gamma_{12}^g + (1-y)z\Gamma_{21}^g + (1-y)(1-z)\Gamma_{11}^g$$

Therefore, the government's average expected payoff (the average fitness) is:

$$\bar{U}^g = x\bar{U}_1^g + (1-x)\bar{U}_2^g$$

Similarly, when the producer adopts the upgraded technology, its expected payoff is:

$$\bar{U}_1^s = z\Gamma_{11}^s + (1-z)\Gamma_{12}^s + h(xE_1 + (1-x)E_2)$$

The expected payoff of the producer maintaining the old technology is:

$$\bar{U}_2^s = z\Gamma_{21}^s + (1-z)\Gamma_{22}^s + h(xE_1 + (1-x)E_2)$$

Therefore, the average expected payoff (the average fitness) of the producer is:

$$\bar{U}^s = y\bar{U}_1^s + (1-y)\bar{U}_2^s$$

For the retailer, the expected payoff of adopting the low-carbon marketing strategy is:

$$\bar{U}_1^r = y\Gamma_{11}^r + (1-y)\Gamma_{21}^r$$

The expected payoff of adopting the non-low-carbon marketing strategy is:

$$\bar{U}_2^r = y\Gamma_{12}^r + (1-y)\Gamma_{22}^r$$

Therefore, the average expected payoff (the average fitness) of the retailer is:

$$\bar{U}^r = z\bar{U}_1^r + (1-z)\bar{U}_2^r$$

Next, we construct the replicated dynamic equation. For the government, its replicated dynamic equation is:

$$\frac{dx}{dt} = x\left(\bar{U}_1^g - \bar{U}^g\right) = x(1-x)\left(\bar{U}_2^g - \bar{U}_1^g\right)$$

For the producer, the replicated dynamic equation is:

$$\frac{dy}{dt} = y\left(\bar{U}_1^s - \bar{U}^s\right) = y(1-y)\left(\bar{U}_2^s - \bar{U}_1^s\right)$$

For the retailer, the replicated dynamic equation is:

$$\frac{dz}{dt} = z\left(\bar{U}_1^r - \bar{U}^r\right) = z(1-z)\left(\bar{U}_2^r - \bar{U}_1^r\right)$$

Therefore, a three-dimensional dynamical system can be constructed by the connection is:

$$\frac{dx}{dt} = x(1-x)\left(\bar{U}_2^g - \bar{U}_1^g\right)$$
$$\frac{dy}{dt} = y(1-y)[z(\Gamma_{11}^s - \Gamma_{12}^s) + (1-z)(\Gamma_{21}^s - \Gamma_{22}^s)]$$
$$\frac{dz}{dt} = z(1-z)[y(\Gamma_{11}^r - \Gamma_{12}^r) + (1-y)(\Gamma_{21}^r - \Gamma_{22}^r)]$$

4.2 Solving the evolutionary game model

Next, we will find the solution of the evolutionary game model. By applying the three-dimensional dynamical system, we find the equilibrium point for the game, as following:

Theorem 1

The equilibrium point of the tripartite game is (x^*, y^*, z^*), where $x^* \in [0,1]$, $y^* \in \{0,1\}$, $z^* \in \{0,1\}$. When $y' \in [0,1]$ and $z' \in [0,1]$, (x', y', z') is also an equilibrium point of the system (in which $x^* \in [0,1]$, $y' = \frac{\Gamma_{22}^s - \Gamma_{21}^s}{(\Gamma_{11}^s - \Gamma_{12}^s) - (\Gamma_{21}^s - \Gamma_{22}^s)}$, $z' = \frac{\Gamma_{22}^r - \Gamma_{21}^r}{(\Gamma_{11}^r - \Gamma_{12}^r) - (\Gamma_{21}^r - \Gamma_{22}^r)}$).

According to the equilibrium point conclusion in Theorem 1, each strategy of the government has its corresponding equilibrium point. Therefore, the government will choose the strategy that maximizes its own payoff, which is $max\{\Gamma_{11}^g, \Gamma_{12}^g, \Gamma_{21}^g \Gamma_{22}^g\}$. Correspondingly, the government's strict strategy will lead to higher carbon emission costs for the producer. The producer will adopt new technologies to reduce carbon emissions costs, which is in line with the purpose of the government's strict strategy.

4.3 Evolutionary Stability Analysis

In reality, due to the inefficiency or lack of transparency of the information, it will almost impossible for the government, the producer and the retailer to directly achieve the optimal strategy or the optimal equilibrium. Therefore, the government, the producer and the retailer, will constantly adjust their strategies according to their vested interests, making dynamic decisions. Whether the system can ultimately reach equilibrium depends on whether the equilibrium point is stable. When all players can reach the final dynamic equilibrium, such strategy is called the Evolutionary Stability Strategy (ESS). We conduct stability analysis as follows:

Theorem 2

When $\lambda < \alpha_1, u < \beta_1$, the evolutionary stability strategy of the system is (X^*, Y_1, Z_1)

When $\lambda > \alpha_2, u < \beta_2$, the evolutionary stability strategy of the system is (X^*, Y_2, Z_1)

When $\lambda < \alpha_2, u > \beta_2$, the evolutionary stability strategy of the system is (X^*, Y_1, Z_2)

When $\lambda > \alpha_2, u > \beta_2$, the evolutionary stability strategy of the system is (X^*, Y_2, Z_2)

In which $X^* \in \{X_1, X_2\}$, $\alpha_1 = \sqrt{\frac{2k_1(2bk_2-u^2)}{k_2}} - bh, \alpha_2 = 2\sqrt{bk_1} - bh, \beta_1 = \sqrt{\frac{4bk_1k_2-k_2(\lambda+bh)^2}{2k_1}}, \beta_2 = \sqrt{2bk_2},$

According to Theorem 2, the stability of equilibrium point among the government, the producer and the retailer will not be affected by whether the government implements tolerant or strict policy. Instead, the consumer's carbon emission reduction sensitive coefficient and the consumer's marketing effort sensitive coefficient affect the stability of the equilibrium point of the system evolution.

Next, we draw phase diagrams of the player's strategies according to Theorem 2.

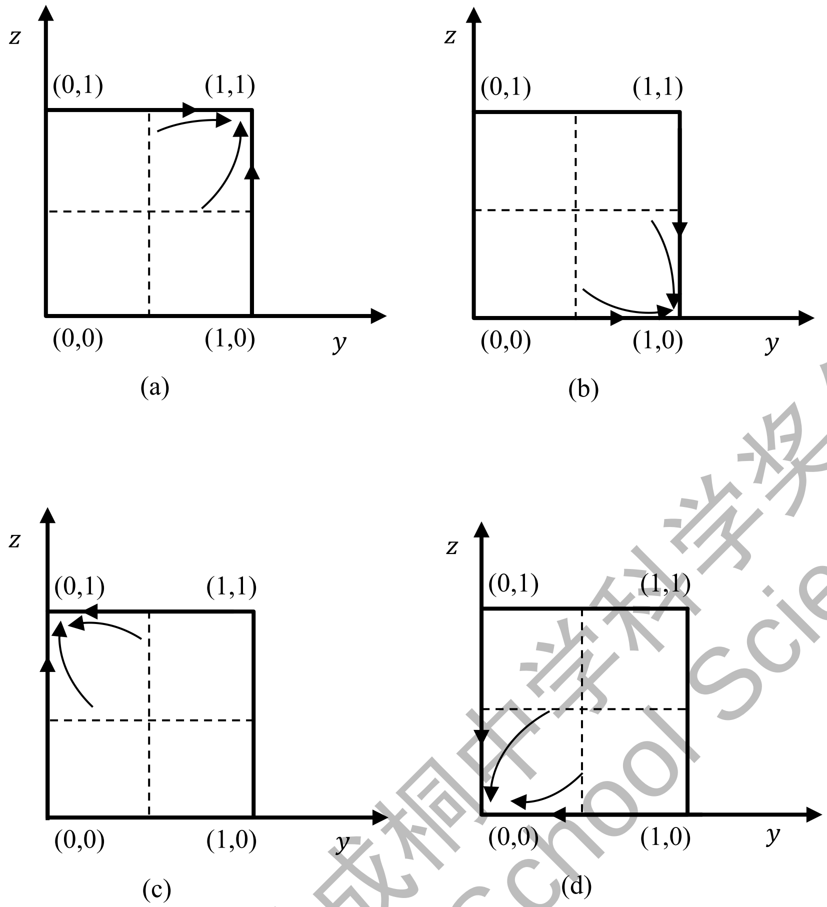

Figure 4.1. Phase diagrams of Producer and Retailer’s strategies

In reality, the government expects the producer to adopt new technologies and the retailer to adopt low-carbon marketing schemes, so the government prefers the result in situation (a) in above Figure 4.1. The strict carbon emission policy adopted by the government will lead to the producer choosing new technologies. Therefore, when the consumer’s sensitivity coefficient to carbon emission and the consumer’s sensitivity coefficient to marketing effort are both small, i.e. $\lambda < \alpha_1, u < \beta_1$, the government should adopt the strict carbon emission policy.

5. Numerical Simulation

Parameter selection:

Assume that in the carbon market, the carbon emission trading price h is linearly related to the probability that the government adopts tolerant policy x : $h = A_1 x + A_2$ $(A_1 < 0)$

5.1 The government's expected payoff and the probability of adopting the tolerant policy

In a low-carbon consumption promoting mechanism, to achieve sustainable economic development and maximize social utility, the government is responsible for the top-level design and the implementation of carbon policies. China has officially annouced its "30-60 Targets". Reducing carbon emissions is not only conducive to promoting the green transformation of economic structure and accelerating the formation of green production modes, but also helps to promote high-quality economic development and brings positive social utility. To achieve a green, low-carbon and sustainable development objective, the government can set carbon emission reduction quotas for producers through the carbon cap-and-trade policy. Given there is the urgency to achieve the objective, the government tends to be more autonomous and goal-oriented in the process of setting carbon quotas. Based on this, we assume that:

Abscissa: the probability of the government adopting the tolerant policy x

Ordinate: the expected payoff of the government $\bar{U}^g$

Relationship: $\bar{U}^g = x\bar{U}_1^g + (1-x)\bar{U}_2^g$

$$\bar{U}_1^g = yz\Gamma_{11}^g + y(1-z)\Gamma_{12}^g + (1-y)z\Gamma_{21}^g + (1-y)(1-z)\Gamma_{22}^g$$

$$\bar{U}_2^g = yz\Gamma_{11}^g + y(1-z)\Gamma_{12}^g + (1-y)z\Gamma_{21}^g + (1-y)(1-z)\Gamma_{22}^g$$

$$\Gamma_{11}^g = \theta\left(T - e_s \cdot q - e_1^3\right) - (e_s \cdot q - e_1^3) - c_1$$

$$\Gamma_{12}^g = \theta\left(T - e_t \cdot q - e_2^3\right) - (e_t \cdot q - e_2^3) - c_1$$

$$\Gamma_{21}^g = \theta\left(T - e_1^2 \cdot q - e_1^3\right) - (e_1^2 \cdot q - e_1^3) - c_1$$

$$\Gamma_{22}^g = \theta\left(T - e_1^2 \cdot q - e_2^3\right) - (e_1^2 \cdot q - e_2^3) - c_1$$

$$e_s = \frac{k_2(\lambda + bh)(a - bc - bhe_0)}{2k_1(2bk_2 - u^2) - k_2(\lambda + bh)^2}, e_t = \frac{(\lambda + bh)(a - bc - bhe_0)}{4bk_1 - (\lambda + bh)^2}$$

$$h = A_1 x + A_2$$

Parameter changes: the probability that producer adopts upgraded technology Y_1 is $y(0 \leq y \leq 1)$, and the probability that retailer adopts low-carbon marketing strategy Z_1 is $z(0 \leq y \leq 1)$

Significance: to find out under what circumstances should the government adopt the tolerant policy, and under what circumstances should it not.

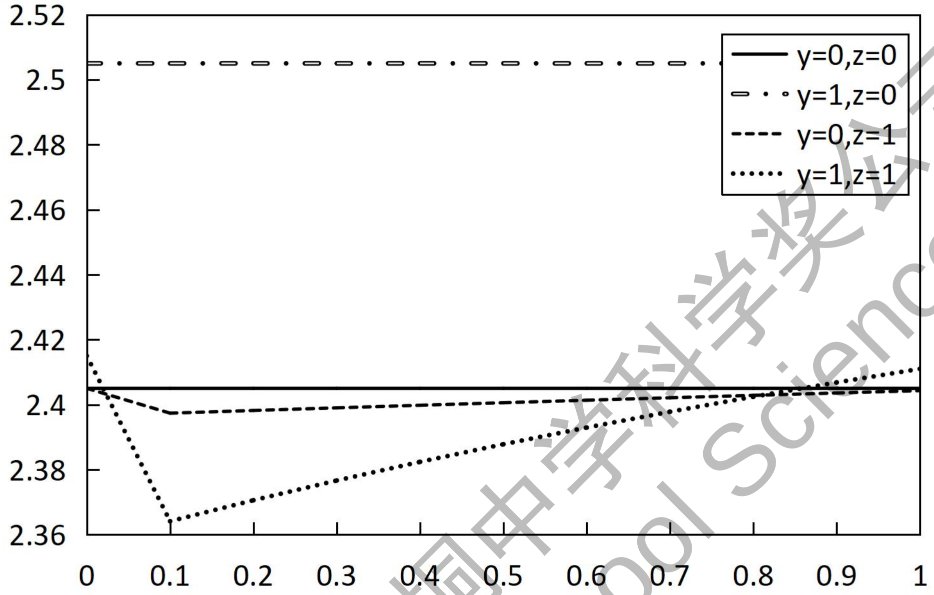

Figure 5.1. The government’s expected payoff

It can be seen from above Figure 5.1 that, by comparing horizontally, when the retailer chooses not to adopt low-carbon marketing strategies, regardless of whether the producer chooses to upgrade the technology, the government obtains the same income no matter it adopts tolerant or strict strategies. When the retailer chooses low-carbon marketing and the producer chooses not to upgrade the technology, the government can obtain greater government payoff by adopting the strict strategy than adopting the tolerant strategy. When the retailer chooses low-carbon marketing and the producer chooses to upgrade the technology, the government can also obtain greater government payoff by adopting the strict strategy than adopting the tolerant strategy.By comparing the results vertically, when the retailer chooses not to conduct low-carbon marketing, the producer can obtain the maximum payoff when it chooses to upgrade the technology.

5.2 The producer’s expected payoff and the probability of upgrading technology

As the supplier of low-carbon emission products, producers play a mediating role between low-carbon productive consumption and non-productive consumption, and communicate the government policy orientation and retailer and consumer behavior. Innovation is the core for producers to carry out low-carbon production strategy, and technological innovation is an important force in improving the efficiency of the producers and essential to form its core competitiveness. In the long run, reducing energy consumption and improving production efficiency helps producers realize economic scale and significantly reduce production costs, bringing a positive impact on the improvement of economic payoffs of producers. However, in the short term, choosing to improve the technology will also increase its costs, and new technologies may be faced with longer research and development cycle, higher input costs, and lower investment return. Based on this, we assume that:

Abscissa: the probability y of updating the technology

Ordinate: the producer's expected payoff $\bar{U}^s$

Relationship: $\bar{U}^s = y\bar{U}_1^s + (1-y)\bar{U}_2^s$

$$\bar{U}_1^s = z\Gamma_{11}^s + (1-z)\Gamma_{12}^s + h(xE_1 + (1-x)E_2)$$

$$\bar{U}_2^s = z\Gamma_{21}^s + (1-z)\Gamma_{22}^s + h(xE_1 + (1-x)E_2)$$

$$\Gamma_{11}^s = \frac{k_1k_2(a-bc-bhe_1^2)^2}{2[2k_1(2bk_2-u^2)-k_2(\lambda+bh)^2]}$$

$$\Gamma_{12}^s = \frac{k_1(a-bc-bhe_1^2)^2}{2[4bk_1-(\lambda+bh)^2]}$$

$$\Gamma_{21}^s = \frac{k_2(a-bc-bhe_1^2)^2}{4(2bk_2-u^2)}$$

$$\Gamma_{22}^s = \frac{(a-bc-bhe_1^2)^2}{8b}$$

$$e_s = \frac{k_2(\lambda+bh)(a-bc-bhe_0)}{2k_1(2bk_2-u^2)-k_2(\lambda+bh)^2}, e_t = \frac{(\lambda+bh)(a-bc-bhe_0)}{4bk_1-(\lambda+bh)^2}$$

Parameter changes: the probability that the government adopts the tolerant strategy X_1 is $x(0 \le x \le 1)$, and the probability that the retailer adopts a low-carbon marketing strategy Z_1 is $z(0 \le z \le 1)$

Significance: to find out when should the producer choose to upgrade the technology and when should it not to.

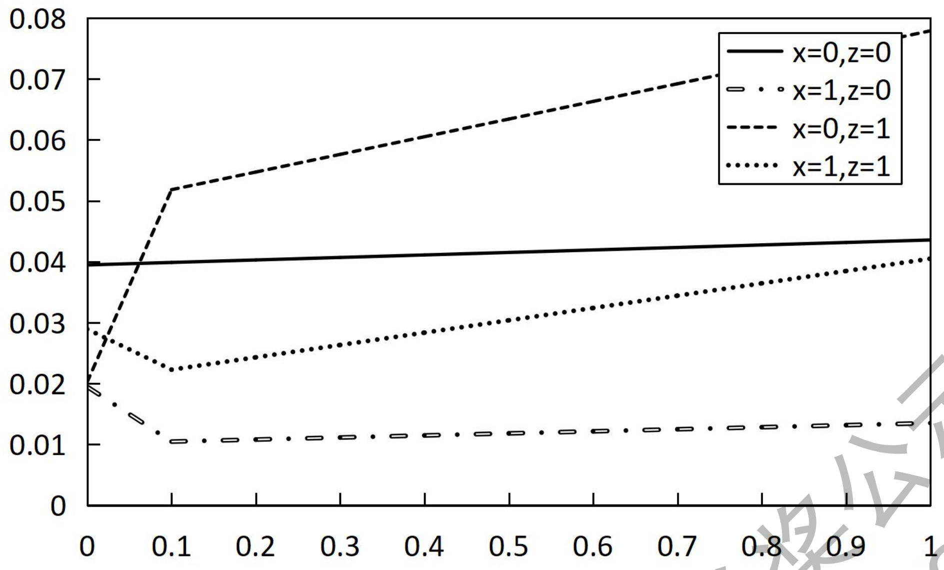

Figure 5.2. The producer's expected payoff

It can be seen from Figure 5.2 that, by comparing horizontally, when the government adopts tolerant policies and the retailer chooses low-carbon marketing, the producer may choose to upgrade technology to obtain the maximum expected payoff. When the government adopts tolerant policies and the retailer chooses not to low-carbon marketing, the producer should choose not to upgrade the technology to obtain the maximum expected payoff. When the government adopts strict policies and the retailer chooses low-carbon marketing, the producer should choose the upgraded technology to significantly improve its expected payoff. When the government adopts strict policies and the retailer chooses not low-carbon marketing, the producer should still choose the upgraded technology to obtain the maximum expected payoff, but the increase is relatively small. Comparing the results of all strategies vertically, when the government adopts strict policies and the retailer chooses low-carbon marketing, the producer should choose to upgrade the technology to obtain the maximum expected payoff.

5.3 The retailer's expected payoff and the probability of low-carbon marketing

With the improvement of their knowledge level and living standard, consumers have gradually fostered the awareness for environmental protection and are becoming increasingly willing to buy green, low-carbon products. However, for consumers, the commodity price is still the core consideration factor when purchasing, and rational consumers in the economic sense are more willing to choose products with lower prices under the same conditions. In most cases, consumers are not the direct beneficiaries of government green subsidies, and the retailer plays an important role in distributing the incentives. When the government distributes green subsidies through retailers, retailers may either transfer the green subsidies to consumers through low-carbon marketing, or they can choose not to market in a low-carbon way and take the consequences. Based on this, we assume that:

Abscissa: probability z of low-carbon marketing

Ordinate: producer's expected payoff $\bar{U}^r$

Relationship: $\bar{U}^r = z\bar{U}_1^r + (1-z)\bar{U}_2^r$

$$\bar{U}_1^r = y\Gamma_{11}^r + (1-y)\Gamma_{21}^r$$

$$\bar{U}_2^r = y\Gamma_{12}^r + (1-y)\Gamma_{22}^r$$

$$\Gamma_{11}^r = \frac{k_1^2 k_2 (2bk_2 - u^2)(a - bc - bhe_1^2)^2}{2[2k_1(2bk_2 - u^2) - k_2(\lambda + bh)^2]^2}$$

$$\Gamma_{12}^r = \frac{bk_1^2 (a - bc - bhe_1^2)^2}{[4bk_1 - (\lambda + bh)^2]^2}$$

$$\Gamma_{21}^r = \frac{k_2 (a - bc - bhe_1^2)^2}{8(2bk_2 - u^2)}$$

$$\Gamma_{22}^r = \frac{(a - bc - bhe_1^2)^2}{16b}$$

$$e_s = \frac{k_2(\lambda + bh)(a - bc - bhe_0)}{2k_1(2bk_2 - u^2) - k_2(\lambda + bh)^2}, e_t = \frac{(\lambda + bh)(a - bc - bhe_0)}{4bk_1 - (\lambda + bh)^2}$$

$$h = A_1 x + A_2$$

Parameter change: the probability that the government adopts the tolerant policy X_1 is $x(0 \le x \le 1)$, and the probability that the producer adopts the upgraded technology Y_1 is $y(0 \le y \le 1)$.

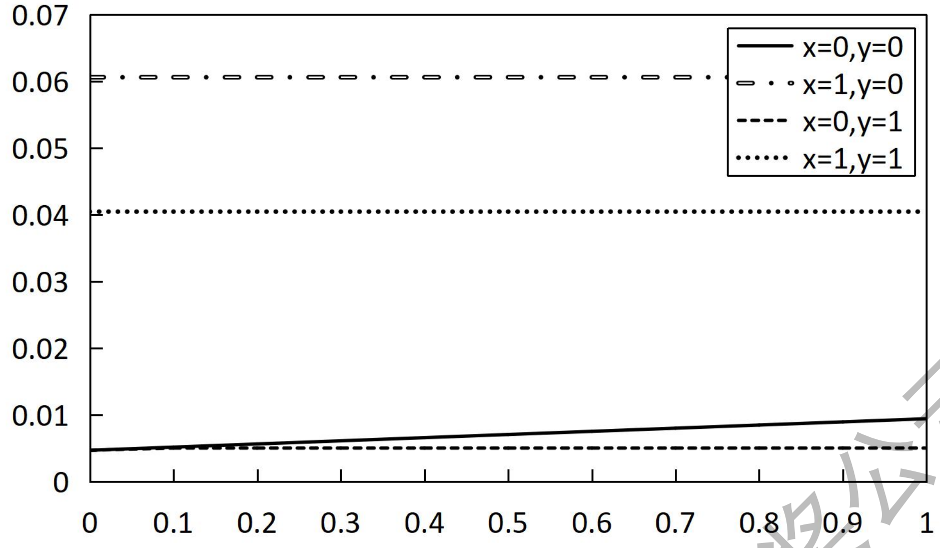

Figure 5.3. The retailer's expected payoff

It can be seen from above Figure 5.3 that, by comparing horizontally, when the government adopts tolerant policies, the expected payoff of the retailer is constant, no matter whether the producer chooses to upgrade technology or not and no matter whether the retailer chooses low carbon marketing or not. When the government chooses tolerant policies and the producer chooses to upgrade technologies, the retailer's expected payoff remains constant, no matter whether the retailer chooses low carbon marketing or not. When the government chooses strict policies and when the producer chooses not to upgrade the technologies, the retailer can obtain the maximum expected payoff by choosing low-carbon marketing. Comparing the results of all strategies vertically, when the government chooses the tolerant policies and the producer chooses not to upgrade the technology, the retailer can obtain the maximum expected payoff.

6. Conclusions

Reducing carbon emission has become a critical measure to address global warming challenges. Low-carbon supply chain plays an important role to achieve governments' carbon emission reduction policy objectives. Existing researches mainly focus on game analysis of two parties, either between the government and the producer, or between upstream and downstream of the supply chain. However, as government plays a leading role by setting carbon emission target, and both the producer and the retailer are key players in the low-carbon supply chain, a tripartite game analysis is meaningful to better understand the real world evolutionary dynamics.

In this paper, we conducted the tripartite game analysis among the government, the producer, and the retailer. The Stackelberg game model is applied to reach the equilibrium point among three parties. Then the Evolutionary game theory is used to achieve the evolutionary stable strategy. Lastly, we conducted a Matlab stimulation study to demonstrate the tripartite evolutionary equilibrium model.

The results show that each of government's carbon emission strategy generates its corresponding equilibrium point, and the strict carbon emission strategy will lead the producer to adopt advanced technologies to reduce carbon emission. In addition, the stability of the equilibrium point among the government, producers and retailers will not be affected by whether the government implements tolerant or strict policy. The sensitivity coefficient of consumers to carbon emission and the sensitivity coefficient of consumers to marketing effort affect the stability of the equilibrium point of the system evolution.

Appreciation Note:

It has been my long time desire to contribute the environment protection. For me, living in a harmonious community with a clean sustainable environment should be the fundamental pursuit of everyone. A severe and acute pneumonia sent me to ICU when I was 5 years old, and through that experience I realized that since the air pollution was one of the reasons that led to the disease that so many people would catch, it should be paid more attention to. After I recovered, I have been participating in multiple environment protection activities, volunteering in various forms of communities services, and publishing articles to promote public awareness. As I grow up, I learned that there are many policies and economic tools already implemented to alleviate the problem, yet more efforts from all aspects of the society may be needed to broader the impacts to protect the environment. The S.-T.Yau High School Science Award provides me this unique opportunity to explore the possibilities and put into practice what I have learned from the school to resolve critical issues.

My appreciation goes to Dr. Jingchun SU. Dr. Su is my advisor, and she guides me for free and understands well on my passion in both the academic studying and environmental protection. When we discussed the research topic selection, she often inspired me to be confident to think big, and think creatively. Dr. Su also advised to apply what I have learned at high school and through extra curricula to understand and improve the existing model, especially the calculus and probability theory. Special thanks also goes to Jin FEI and Chaoyang CHENG, both have given me great advice and supports in the modeling process as well as conducting the stimulation work. All errors are mine.

It has been a great learning experience. I know that this paper is far from good, and there are a lot of rooms to improve. I will continue to build my skills and knowledge in financial modeling and game theory study, and to further expand the research around this paper's topic.

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Appendix I

Combination of behavior strategies, in details

Combination of Behavior strategies	Government payoff	Producer payoff	Retailer payoff
(X_1, Y_1, Z_1)	$\theta(T - e_s \cdot q - e_1^3) - (e_s \cdot q - e_1^3) - c_1$	$\frac{k_1 k_2 (a-bc-bhe_1^2)^2}{2[2k_1(2bk_2-u^2)-k_2(\lambda+bh)^2]} + hE_1$	$\frac{k_1^2 k_2 (2bk_2-u^2)(a-bc-bhe_1^2)^2}{2[2k_1(2bk_2-u^2)-k_2(\lambda+bh)^2]^2}$
(X_1, Y_1, Z_2)	$\theta(T - e_t \cdot q - e_2^3) - (e_1^2 \cdot q - e_2^3) - c_1$	$\frac{k_1(a-bc-bhe_1^2)^2}{2[4bk_1-(\lambda+bh)^2]} + hE_1$	$\frac{bk_1^2(a-bc-bhe_1^2)^2}{[4bk_1-(\lambda+bh)^2]^2}$
(X_1, Y_2, Z_1)	$\theta(T - e_1^2 \cdot q - e_1^3) - (e_1^2 \cdot q - e_1^3) - c_1$	$\frac{k_2(a - bc - bhe_1^2)^2}{4(2bk_2 - u^2)} + hE_1$	$\frac{k_2(a - bc - bhe_1^2)^2}{8(2bk_2 - u^2)}$
(X_1, Y_2, Z_2)	$\theta(T - e_1^2 \cdot q - e_2^3) - (e_1^2 \cdot q - e_2^3) - c_1$	$\frac{(a - bc - bhe_1^2)^2}{8b} + hE_1$	$\frac{(a - bc - bhe_1^2)^2}{16b}$
(X_2, Y_1, Z_1)	$\theta(T - e_s \cdot q - e_1^3) - (e_s \cdot q - e_1^3) - c_1$	$\frac{k_1 k_2 (a-bc-bhe_1^2)^2}{2[2k_1(2bk_2-u^2)-k_2(\lambda+bh)^2]} + hE_2$	$\frac{k_1^2 k_2 (2bk_2-u^2)(a-bc-bhe_1^2)^2}{2[2k_1(2bk_2-u^2)-k_2(\lambda+bh)^2]^2}$
(X_2, Y_1, Z_2)	$\theta(T - e_t \cdot q - e_2^3) - (e_t \cdot q - e_2^3) - c_1$	$\frac{k_1(a - bc - bhe_1^2)^2}{2[4bk_1 - (\lambda + bh)^2]} + hE_2$	$\frac{bk_1^2(a-bc-bhe_1^2)^2}{[4bk_1-(\lambda+bh)^2]^2}$
(X_2, Y_2, Z_1)	$\theta(T - e_2^2 \cdot q - e_1^3) - (e_2^2 \cdot q - e_1^3) - c_1$	$\frac{k_2(a-bc-bhe_1^2)^2}{4(2bk_2-u^2)} + hE_2$	$\frac{k_2(a-bc-bhe_1^2)^2}{8(2bk_2-u^2)}$
(X_2, Y_2, Z_2)	$\theta(T - e_2^2 \cdot q - e_2^3) - (e_2^2 \cdot q - e_2^3) - c_1$	$\frac{(a-bc-bhe_1^2)^2}{8b} + hE_2$	$\frac{(a-bc-bhe_1^2)^2}{16b}$