Student: Yilin Jiang

Beijing, China

Advisor: Jingchun SU

A Tripartite Game Analysis on Evolutionary Equilibrium of Carbon Emission

September 15, 2022

A Tripartite Game Analysis on Evolutionary Equilibrium of Carbon Emission

Yilin Jiang

Abstract: Governments implement various policy tools to achieve carbon emission reduction objectives. These policy objectives can only be achieved through impacting the producer and retailer, two key players of the low-carbon supply chain. This paper studies the tripartite game approach among the government, the producer, and the retailer. Firstly, the game model is applied to reach the equilibrium point among three parties. Then the Evolutionary game theory is used to achieve the evolutionary stable strategy. Finally, a simulation study is conducted to illustrate how the tripartite evolutionary equilibrium model would work. The results show that each of government's carbon emission strategy generates its corresponding equilibrium point, and a strict carbon emission strategy will lead the producer to adopt advanced technologies to reduce carbon emission. In addition, the stability of the equilibrium among the government, the producer and the retailer will not be affected by the government's policy choice. The consumer's sensitivity coefficient to carbon emission and the consumer's sensitivity coefficient to the retailer's marketing effort affect the stability of the equilibrium point of the system evolution.

Keywords: Carbon Emission, Low-carbon Supply Chain, Tripartite Game, Evolutionary Stable Strategy

Index:

1. Introduction

Due to global warming and environmental degradation, reducing the amount of carbon emissions caused by traditional energy consumption has become a global objective. Reducing carbon emissions is not only to protect the homeland on which human beings live, but also to guarantee the sustainable development of social economy. Many countries have started taking actions. Some European countries such as Germany has promised to ban gasoline and diesel cars from 2030 and reduce its carbon footprint by 80 to 95 percent by 2050. China officially initiated the pilot program of carbon emission trading in 2011, and launched a nationwide carbon trading market in July 2021. In September 2020, Chinese formally proposed to target reaching a carbon peak by 2030 and a carbon neutrality by 2060, A.K.A the "30-60 Target".

Governments around the world have been trying to cut carbon emissions of high-polluting industries through implementing carbon policies. Building the low-carbon supply chain has increasingly become an effective measure to achieve the government's policy objective, especially when it is combined with economic incentives. A low-carbon supply chain can not only reduce the damage to the environment, but also optimize the resource allocation, eventually achieving a synchronous development of social economy and the protection of environment.

Achieving low-carbon supply chain starts with the government implementing carbon emissions abatement policies, which includes: levy carbon tax, set carbon quota, carry out carbon trade, implement green electricity premium and others. These policies can only achieve their goals by effectively influencing the producers and retailers (or distributors) of products. At the same time, education and publicity should be strengthened to cultivate consumers' awareness of environmental conservation, so as to affect the supply side from the demand perspective. Typically, government's carbon emission targets do not directly affect individual consumers, but rather do so indirectly through the distribution of products (retailers), such as by providing green subsidies or coupons.

Due to the competing interests among enterprises, they would react differently toward the government's policies, resulting in a game between the government and the producer, the producer and the retailer, as well as the retailer and the government (collectively, "the three parties"). A tripartite game model will be important to understand how the government's carbon emission policies will affect the producer and the retailer, and eventually achieve its policy objective.

In this paper, the game model is mainly used to analyze the game approach among the three parties, of which the government is in the leading position. However, in the real world, due to reasons such as lack of information transparency or inefficiency of policy communication, it is difficult for the three parties to obtain the optimal strategy or optimal equilibrium directly. They will constantly adjust their own strategies based on vested interests and make decisions dynamically. As such, the Evolutionary game theory is also applied to further analyze the evolutionary stability strategy. In the final part, the Matlab tool is applied to simulate the theory and analysis, illustrating how this game theory would perform in the real world.

Existing studies mainly focus on the game analysis between two parties, either between the government and the producer, or between upstream and downstream supply chain. Wang and Cheng (2021) conducted game studies on the producer's emission reduction and the retailer's low-carbon marketing strategies. In their study, the game model and the evolutionary stability strategy are applied. However, there is a lack of analysis and theoretical model that includes the government, the producer and the retailer to a tripartite game system. This paper leveraged existing researches in this field, including the model approach, certain key assumptions and conclusions conducted by Wang and Cheng (2021), but expanded to a government leading tripartite game study. The game strategy matrix expands from four to eight scenarios, and the simulation also covers more complicated but closer-to-reality tripartite dynamic evolution. The focus of this paper is to bring the government into this game analysis and to understand its real-world implication, rather than pure mathematical-oriented model construction.

The findings from this project illustrate that the government's carbon emission policy will affect the producer and the retailer's actions accordingly, and that it is important to educate consumers and promote public awareness towards the low-carbon emission approach.

2. Literature Review

Typically, government's carbon reduction policies include levying carbon tax, setting carbon quota, carrying out carbon trading system, implementing green electricity premium and some others. Among them, carbon tax was first implemented in Northern European countries in the early 1990s, and has now been implemented in most developed countries and a few developing countries. Ye et al. (2022) found that implementing carbon tax separates it from other resource taxes like consumption tax, which would make it easier for the tax authorities to verify the credibility of the amount of the carbon emissions reported by enterprises themselves. However, it is still difficult to completely avoid under-reporting, making the policy less effective. Zhang et al. (2022) studied that the government would try to carry out the carbon trading system as another method, which facilitates the development of green finance. In addition, Jiang and Sun (2016) studied that the government may set carbon quota and issue carbon credits according to the amount of the carbon emission submitted by producers. However, due to information gaps, it is difficult for the government to verify the real amount of carbon emissions, and therefore providing more credits than actually needed. Due to such uncertainty, the allocation of carbon credits would eventually be decided by games between the government and the enterprises.

The government's influence over the low-carbon supply chain has been thoroughly studied. Most researches indicate that the government needs to carry out differential pricing mechanism according to the enterprise scale, in order to maximize economical efficiency. Zhang et al. (2022) used the projection contraction algorithm to solve the mathematical model of closed-loop supply chain satisfying Nash equilibrium, and concluded that in the context of multiple demand markets and one carbon trading market, the government should set corresponding carbon allowances according to the business scope of enterprises. Yuan et al. (2022) constructed the pricing and decision-making model of the recovery supply chain by combining the carbon allowances allocated to enterprises by the government. By comparing the corresponding pricing changes of enterprises under different carbon allowances, they calculated the appropriate amount of carbon allowances that the government should offer enterprises. Li et al.(2020) established a closed-loop supply chain composed of the government, automobile producers and automobile recyclers, and discussed the incentive mechanism of manufacturing enterprises to their downstream recyclers under the mechanism of carbon trading market.

A large amount of literatures have studied the decision-making process and result in the supply chain based on existing policies. Xia et al. (2021) studied the pricing game between producers and retailers based on the cross-shareholding strategy, and verified the pricing rule of retailers by analyzing the changing trend of the cross-shareholding ratio on the profit value of producers and retailers under different carbon trading prices. Diao et al. (2021) studied the pricing game process between producers and retailers for products with strong substitutions in a competitive environment, and proposed the use of payoff sharing contract to reasonably distribute the profits of both parties. Liao et al. (2021) discussed the emission reduction decision of energy saving service companies and their upstream and downstream enterprises under the carbon trading policy. Their research shows that the emission reduction input on the supply chain has an inverse relationship with energy saving efficiency.

Some scholars performed innovative studies by jumping away from the traditional policy-oriented approach. Geels et al. (2017) put forward the "socio-technical" framework theory, hoping to constrain social groups to accelerate low-carbon transformation through mutual collaboration and influence. Their approach provides a new angel to the low-carbon policy research by take the social group into consideration. Wang and Cheng (2021) conducted game studies on the producer's emission reduction and the retailer's low-carbon marketing strategies. Their study found that raising consumers' environmental awareness helps the government to promote carbon emission reduction.

Previous studies focused mainly on two parties game analysis, either the game between the government and the producer, or the game between the producer and the retailer. The innovation of this paper lies in including the government, the producer and the retailer to create a three-party game, which provides a thorough and close-loop discussion on the game in the entire supply chain. The game strategy matrix expand from four to eight scenarios. Furthermore, most previous studies aimed at finding the equilibrium point of the game. However, in practice, producers and retailers may perform with bounded rationality and thus need trial and error before deciding on whether or not to reduce emissions and market in low-carbon ways. Therefore, the theoretical equilibrium might not be easily achieved. In this paper, we use the evolution stability strategy to evaluate the stability of the equilibrium point, which has great practical significance.

In this paper, we focus on finding the optimum solution in the government-producer-retailer game using the game theory and evaluating the stability of the equilibrium point using the the evolutionary stability strategy. A simulation study is conducted to illustrate how the tripartite evolutionary equilibrium model would work.

3. Evolutionary Game Model Description and Assumptions

3.1 Model introduction

Existing research mainly focuses on the coordination and decision-making of the supply chain under the circumstance of the carbon cap-and-trade policy, and the game of emission reduction between enterprises and the government, and between upstream and downstream enterprises. There is a lack of theoretical model that includes the government, the producer and the retailer to a three-parties game system. Due to the fact that central and local governments are responsible for the management of the carbon market, and the government oversees the carbon market, it is necessary to regard the government as a main part of the game. Meanwhile, supply chains composed by producers and retailers are critical to carry out governments' policy objectives. This article takes the tripartite chain consisting of the government plus a single producer and a single retailer as the research object, so as to study the game among the producer's decision on upgrading technology ("new technology") and the retailer's decision on adopting low-carbon marketing under the government's different carbon policy approaches.

A tripartite system consisting of a government G, a producer S and a retailerR is the studying objective. The decision makers in this system are all bounded rational, and it is difficult to make an optimal choice in one decision-making. To obtain a stable equilibrium, it is necessary to adjust the strategy constantly. In addition to the original operation decision, the strategy set of the government is $S_1 = \{X_1, X_2\}$. X_1 represents " tolerant policy", which indicates a higher carbon allowance（E_1）allocated by the government to producers, X_2 represents "strict policy", indicating a lower carbon allowance（E_2，$E_2 < E_1$）allocated by the government to the producer; the strategy set of the producer is $S_2 = \{Y_1, Y_2\}$. Y_1 represents adopting new technology, and Y_2 represents adopting traditional technology; the unit carbon emission of new technology is e_1^2, and the unit emission of traditional technology is e_2^2（$e_1^2 < e_2^2$）. There is a cost to upgrade technology. The strategy set of the retailer is $S_3 = \{Z_1, Z_2\}$. Z_1 represents low-carbon marketing, and Z_2 represents non-low carbon marketing; low carbon emission is e_1^3, and non-low carbon emission is e_2^3（$e_1^3 < e_2^3$）.

3.2 Hypothesis and notation

Hypothesis 1 Suppose that a simple linear relationship exists between the demand function and the retail price of the product, and the producer's emission reduction efforts and the retailer's marketing efforts, that is, $D = a - bp + \lambda(e_1^2 - e_2^2) + \mu g$, where, a represents the market size of the product, b represents consumer price sensitivity coefficient, λ represents consumer carbon emission reduction sensitivity coefficient, and μ represents consumer marketing effort sensitivity coefficient.

Hypothesis 2 When the producer upgrades technology, the carbon emission per unit product is e_1^2. When the producer adopts traditional technology, the carbon emission per unit product is e_2^2, and $(e_2^2 - e_1^2)$ represents the unit product reduction in carbon emissions.

Hypothesis 3 When the producer upgrades technology, the carbon emission per unit product is reduced by $(e_2^2 - e_1^2)$, and the cost of emission reduction effort is $\frac{1}{2}k_1(e_2^2 - e_1^2)$. Producer profit = production income - cost of updating technology – cost of purchasing carbon (if technology is upgraded, then there is a cost; if technology is not upgraded, then there is no cost), that is, $u_2(x,y,z) = u_2^R(x,y,z) - u_2^T(x,y,z) - u_2^c(x,y,z)$, where the production income is $u_2^R(x,y,z) = (\omega - c)q$, and the cost of purchasing carbon is $u_2^c(x,y,z) = h(e_i^2 q - E_1)$ or $h(e_i^2 q - E_2)$.

Hypothesis 4 When the retailer adopts low-carbon marketing, the carbon emissions per unit product will be reduced by $(e_2^3 - e_1^3)$. g represents the retailer's marketing effort, and the marketing effort cost is $\frac{1}{2}k_2 g^2$, Retailer's payoff = Retailer's income - cost of low-carbon, i.e. $u_3(x,y,z) = u_3^R(x,y,z) - u_3^C(x,y,z)$, where the retailer income is $u_3^R(x,y,z) = (p - \omega)q$

Hypothesis 5 When the government chooses X_1 " tolerant policy", the carbon allowance （E_1） allocated by the government to producers is higher, and when X_2 "strict policy" is selected, the allowance （E_2，$E_2 < E_1$） allocated by the government to the producer is lower. The policy cost the government needs to pay is $u_1^C(x,y,z) = c_1$, and the government's payoff = social benefits - carbon emissions - policy costs, that is, $u_1(x,y,z) = u_1^S(x,y,z) - u_1^{CE}(x,y,z) - u_1^C(x,y,z)$

Hypothesis 6 Suppose that social benefits depend on the overall reduction in carbon emissions, that is, $u_1^S(x,y,z) = \theta(T - T(y,z))$, where θ represents the government's payoff constant, and $T(Y_i, Z_j)$ is carbon emissions of the producer and the retailer $T(Y_i, Z_j) = e_i^2 \cdot q + e_j^3$

Hypothesis 7 Define carbon emissions: $u_1^{CE}(x,y,z) = T(Y_i, Z_j) = e_i^2 \cdot q + e_j^3$

Table 1. Meaning of parameters

Parameters	Parameter Meaning
θ	Government payoff constant
T	Total amount of carbon allowances currently available in the consumer carbon trading market
$T(Y_i, Z_j)$	Carbon emissions of the producer and the retailer
ω	Wholesale price per unit
p	Retail price per unit
q	Product order（suppose $q = D$），$q > \omega$
D	Demand
e_1^2	Carbon emissions per unit when adopting new technology
e_2^2	Carbon emissions per unit when adopting traditional technology
e_1^3	Carbon emissions per unit when adopting low-carbon marketing
e_2^3	Carbon emissions per unit when adopting non-low-carbon marketing
k_1, k_2	Cost constant
E_1	Carbon allowances when adopting “tolerant policy”
E_2	Carbon allowances when adopting "strict policy"
h	The trading price of carbon emissions per unit
g	Retailer's marketing effort
a	Market size of the product
b	Consumer’s price sensitivity coefficient
λ	Consumer’s carbon emission reduction sensitivity coefficient
μ	Consumer’s marketing effort sensitivity coefficient

3.3 Payoff matrix of game subject's strategy

From the behavioral strategy sets of the government, the producer and the retailer, eight behavioral strategies for the evolutionary game could be derived (as shown in Table 2).

With the model parameters shown in Table 1, the payoffs of the government, the producer and the retailer under eight behavioral strategies could be obtained. See Table 2 below for details.

Table 2. Payoff matrix

Behavioral Strategy	Government's Payoff	Producer's Payoff	Retailer's Payoff
(X_1, Y_1, Z_1)	$\theta(T - e_1^2 \cdot q - e_1^3) - (e_1^2 \cdot q - e_1^3) - c_1$	$(\omega - c)q - \frac{1}{2}k_1(e_2^2 - e_1^2) - h(e_1^2 q - E_1)$	$(p - \omega)q - \frac{1}{2}k_2 g^2$
(X_1, Y_1, Z_2)	$\theta(T - e_1^2 \cdot q - e_2^3) - (e_1^2 \cdot q - e_2^3) - c_1$	$(\omega - c)q - \frac{1}{2}k_1(e_2^2 - e_1^2) - h(e_1^2 q - E_1)$	$(p - \omega)q$
(X_1, Y_2, Z_1)	$\theta(T - e_2^2 \cdot q - e_1^3) - (e_2^2 \cdot q - e_1^3) - c_1$	$(\omega - c)q - h(e_2^2 q - E_1)$	$(p - \omega)q - \frac{1}{2}k_2 g^2$
(X_1, Y_2, Z_2)	$\theta(T - e_2^2 \cdot q - e_2^3) - (e_2^2 \cdot q - e_2^3) - c_1$	$(\omega - c)q - h(e_2^2 q - E_1)$	$(p - \omega)q$
(X_2, Y_1, Z_1)	$\theta(T - e_1^2 \cdot q - e_1^3) - (e_1^2 \cdot q - e_1^3) - c_1$	$(\omega - c)q - \frac{1}{2}k_1(e_2^2 - e_1^2) - h(e_1^2 q - E_2)$	$(p - \omega)q - \frac{1}{2}k_2 g^2$
(X_2, Y_1, Z_2)	$\theta(T - e_1^2 \cdot q - e_2^3) - (e_1^2 \cdot q - e_2^3) - c_1$	$(\omega - c)q - \frac{1}{2}k_1(e_2^2 - e_1^2) - h(e_1^2 q - E_2)$	$(p - \omega)q$
(X_2, Y_2, Z_1)	$\theta(T - e_2^2 \cdot q - e_1^3) - (e_2^2 \cdot q - e_1^3) - c_1$	$(\omega - c)q - h(e_2^2 q - E_2)$	$(p - \omega)q - \frac{1}{2}k_2 g^2$
(X_2, Y_2, Z_2)	$\theta(T - e_2^2 \cdot q - e_1^3) - (e_2^2 \cdot q - e_2^3) - c_1$	$(\omega - c)q - h(e_2^2 q - E_2)$	$(p - \omega)q$

3.4 Construction of evolutionary game model

To achieve sustainable development and carbon emission reduction, the government released a set of carbon emission reduction policies, such as carbon cap-and-trade policy, carbon tax, etc. This article focuses on the decision-making of various players under the carbon cap-and-trade policy. The government gives producers a carbon emission quota, only when producers keep their carbon emissions lower than this quota, may they sell the remaining carbon emissions in the carbon market. On the other hand, if the producer's carbon emission exceeds this quota, the producer has the option to purchase carbon emission in the market. At the same time, since the government needs to consider the impact of the carbon cap-and-trade policy on the social and economic benefits as a whole, it will choose between tolerant and strict policies according to actual economic conditions. On this base, we first determine the equilibrium point between the producer and the retailer under various government strategies, followed by finding the optimal government strategy. Lastly, we will analyze the stability of the equilibrium point according to the evolutionary game model.

In this case, firstly, we need to analyze what is the optimal strategy among the government, the producer, and the retailer, which involves the concept of equilibrium point. However, in fact, due to factors such as information differences, it is impossible for the government, the producer, and the retailer to directly obtain the optimal strategy or optimal equilibrium. They constantly adjust their strategies based on vested interests and make decisions dynamically. Whether the system can reach equilibrium depends on whether the equilibrium point is stable. Therefore, we will dedicate a separate chapter to the stability analysis.

4. Evolutionary Game Model Construction and Stability Analysis

Under the above model assumptions, we establish an evolutionary game model to further analyze the problem. First, we need to analyze the optimal strategy between the government, the producer, and the retailer, which involves finding the equilibrium point of the game. Next, we construct the evolutionary game model.

4.1 The construction of the evolutionary model

According to Wang and Cheng (2021), in accordance with the existing carbon emission policies, once the producer has determined the wholesale price per unit and the emissions of the new technology, and once the retailer has determined the retail price, the income matrix between players is decided, as shown in the following table (see Appendix I for detailed formula):

Table 3. Combination of behavior strategies

Behavior Strategies	Government's Payoff	Producer's Payoff	Retailer's Payoff
(X_1, Y_1, Z_1)	Γ^g_{11}	$\Gamma^s_{11} + hE_1$	Γ^r_{11}
(X_1, Y_1, Z_2)	Γ^g_{12}	$\Gamma^s_{12} + hE_1$	Γ^r_{12}
(X_1, Y_2, Z_1)	Γ^g_{21}	$\Gamma^s_{21} + hE_1$	Γ^r_{21}
(X_1, Y_2, Z_2)	Γ^g_{22}	$\Gamma^s_{22} + hE_1$	Γ^r_{22}
(X_2, Y_1, Z_1)	Γ^g_{11}	$\Gamma^s_{11} + hE_2$	Γ^r_{11}
(X_2, Y_1, Z_2)	Γ^g_{12}	$\Gamma^s_{12} + hE_2$	Γ^r_{12}
(X_2, Y_2, Z_1)	Γ^g_{21}	$\Gamma^s_{21} + hE_2$	Γ^r_{21}
(X_2, Y_2, Z_2)	Γ^g_{22}	$\Gamma^s_{22} + hE_2$	Γ^r_{22}

Based on the above, we will construct the evolutionary game model. We try to construct a complex dynamic equation of three parties in this paper. We assume that the probability of the government adopting the tolerant strategy X_1 is $x(0 \le x \le 1)$, the probability of the producer adopting the new technology Y_1 is $y(0 \leq y \leq 1)$, and the probability of the retailer adopting the low-carbon marketing strategy Z_1 is $z(0 \leq z \leq 1)$.

For the government, the expected payoff from the tolerant strategy is:

$$\bar{U}_1^g = yz\varGamma_{11}^g + y(1-z)\varGamma_{12}^g + (1-y)z\varGamma_{21}^g + (1-y)(1-z)\varGamma_{11}^g$$

The expected payoff from adopting a strict strategy is:

$$\bar{U}_2^g = yz\varGamma_{11}^g + y(1-z)\varGamma_{12}^g + (1-y)z\varGamma_{21}^g + (1-y)(1-z)\varGamma_{11}^g$$

Therefore, the government's average expected payoff (the average fitness) is:

$$\bar{U}^g = x\bar{U}_1^g + (1-x)\bar{U}_2^g$$

Similarly, when the producer adopts the upgraded technology, its expected payoff is:

$$\bar{U}_1^s = z\varGamma_{11}^s + (1-z)\varGamma_{12}^s + h(xE_1 + (1-x)E_2)$$

The expected payoff of the producer maintaining the old technology is:

$$\bar{U}_2^s = z\varGamma_{21}^s + (1-z)\varGamma_{22}^s + h(xE_1 + (1-x)E_2)$$

Therefore, the average expected payoff (the average fitness) of the producer is:

$$\bar{U}^s = y\bar{U}_1^s + (1-y)\bar{U}_2^s$$

For the retailer, the expected payoff of adopting the low-carbon marketing strategy is:

$$\bar{U}_1^r = y\varGamma_{11}^r + (1-y)\varGamma_{21}^r$$

The expected payoff of adopting the non-low-carbon marketing strategy is:

$$\bar{U}_2^r = y\varGamma_{12}^r + (1-y)\varGamma_{22}^r$$

Therefore, the average expected payoff (the average fitness) of the retailer is:

$$\bar{U}^r = z\bar{U}_1^r + (1-z)\bar{U}_2^r$$

Next, we construct the replicated dynamic equation. For the government, its replicated dynamic equation is:

$$\frac{dx}{dt} = x\left(\bar{U}_1^g - \bar{U}^g\right) = x(1-x)\left(\bar{U}_2^g - \bar{U}_1^g\right)$$

For the producer, the replicated dynamic equation is:

$$\frac{dy}{dt} = y\left(\bar{U}_1^s - \bar{U}^s\right) = y(1-y)\left(\bar{U}_2^s - \bar{U}_1^s\right)$$

For the retailer, the replicated dynamic equation is:

$$\frac{dz}{dt} = z\left(\bar{U}_1^r - \bar{U}^r\right) = z(1-z)\left(\bar{U}_2^r - \bar{U}_1^r\right)$$

Therefore, a three-dimensional dynamical system can be constructed by the connection is:

$$\frac{dx}{dt} = x(1-x)\left(\bar{U}_2^g - \bar{U}_1^g\right)$$
$$\frac{dy}{dt} = y(1-y)[z(\Gamma_{11}^s - \Gamma_{12}^s) + (1-z)(\Gamma_{21}^s - \Gamma_{22}^s)]$$
$$\frac{dz}{dt} = z(1-z)[y(\Gamma_{11}^r - \Gamma_{12}^r) + (1-y)(\Gamma_{21}^r - \Gamma_{22}^r)]$$

4.2 Solving the evolutionary game model

Next, we will find the solution of the evolutionary game model. By applying the three-dimensional dynamical system, we find the equilibrium point for the game, as following:

Theorem 1

The equilibrium point of the tripartite game is (x^*, y^*, z^*), where $x^* \in [0,1]$, $y^* \in \{0,1\}$, $z^* \in \{0,1\}$. When $y' \in [0,1]$ and $z' \in [0,1]$, (x', y', z') is also an equilibrium point of the system (in which $x^* \in [0,1]$, $y' = \frac{\Gamma_{22}^s - \Gamma_{21}^s}{(\Gamma_{11}^s - \Gamma_{12}^s) - (\Gamma_{21}^s - \Gamma_{22}^s)}$, $z' = \frac{\Gamma_{22}^r - \Gamma_{21}^r}{(\Gamma_{11}^r - \Gamma_{12}^r) - (\Gamma_{21}^r - \Gamma_{22}^r)}$).

According to the equilibrium point conclusion in Theorem 1, each strategy of the government has its corresponding equilibrium point. Therefore, the government will choose the strategy that maximizes its own payoff, which is $max\{\Gamma_{11}^g, \Gamma_{12}^g, \Gamma_{21}^g \Gamma_{22}^g\}$. Correspondingly, the government's strict strategy will lead to higher carbon emission costs for the producer. The producer will adopt new technologies to reduce carbon emissions costs, which is in line with the purpose of the government's strict strategy.

4.3 Evolutionary stability analysis

In reality, due to the inefficiency or lack of transparency of the information, it will be almost impossible for the government, the producer and the retailer to directly achieve the optimal strategy or the optimal equilibrium. Therefore, the government, the producer and the retailer will constantly adjust their strategies according to their vested interests, making dynamic decisions. Whether the system can ultimately reach equilibrium depends on whether the equilibrium point is stable. When all players can reach the final dynamic equilibrium, such a strategy is called the Evolutionary Stability Strategy (ESS). We conduct stability analysis as follows:

Theorem 2

When $\lambda < \alpha_1, u < \beta_1$, the evolutionary stability strategy of the system is (X^*, Y_1, Z_1)
When $\lambda > \alpha_2, u < \beta_2$, the evolutionary stability strategy of the system is (X^*, Y_2, Z_1)
When $\lambda < \alpha_2, u > \beta_2$, the evolutionary stability strategy of the system is (X^*, Y_1, Z_2)
When $\lambda > \alpha_2, u > \beta_2$, the evolutionary stability strategy of the system is (X^*, Y_2, Z_2)

In which $X^* \in \{X_1, X_2\}$, $\alpha_1 = \sqrt{\frac{2k_1(2bk_2 - u^2)}{k_2}} - bh, \alpha_2 = 2\sqrt{bk_1} - bh, \beta_1 = \sqrt{\frac{4bk_1k_2 - k_2(\lambda + bh)^2}{2k_1}}, \beta_2 = \sqrt{2bk_2},$

According to Theorem 2, the stability of equilibrium point among the government, the producer and the retailer will not be affected by whether the government implements tolerant or strict policy or not. Instead, the consumer's carbon emission reduction sensitivity coefficient and the consumer's marketing effort sensitivity coefficient affect the stability of the equilibrium point of the system evolution.

Next, we draw phase diagrams of the player's strategies according to Theorem 2.

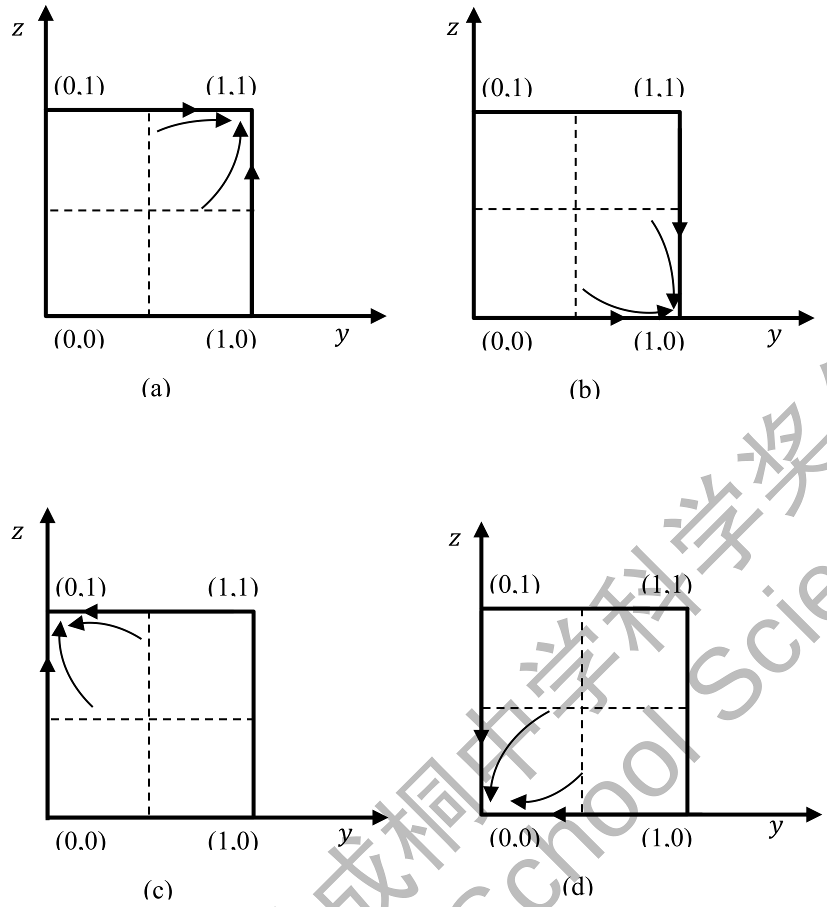

Figure 4.1. Phase diagrams of the producer and the retailer’s strategies

In reality, the government expects the producer to adopt new technologies and the retailer to adopt low-carbon marketing schemes, so the government prefers the result in situation (a) in above Figure 4.1. The strict carbon emission policy adopted by the government will lead to the producer choosing new technologies. Therefore, when the consumer’s sensitivity coefficient to carbon emission and the consumer’s sensitivity coefficient to marketing effort are both small, i.e. $\lambda < \alpha_1, u < \beta_1$, the government should adopt the strict carbon emission policy.

5. Numerical Simulation

Parameter selection:

Assume that in the carbon market, the carbon emission trading price h is linearly related to the probability that the government adopts tolerant policy x : $h = A_1 x + A_2$ $(A_1 < 0)$

5.1 The government's expected payoff and the probability of adopting the tolerant policy

In a low-carbon consumption promoting mechanism, to achieve sustainable economic development and maximize social utility, the government is responsible for the top-level design and the implementation of carbon policies. China has officially announced its "30-60 Target". Reducing carbon emissions is not only conducive to promote the green transformation of economic structure and accelerating the formation of green production modes, but also meaningful to promote high-quality economic development and brings positive social utility. To achieve a green, low-carbon and sustainable development objective, the government can set carbon emission reduction quotas for producers through the carbon cap-and-trade policy. Given that there is the urgency to achieve the objective, the government tends to be more autonomous and goal-oriented in the process of setting carbon quotas. Based on this, we assume that:

Abscissa: the probability of the government adopting the tolerant policy x

Ordinate: the expected payoff of the government $\bar{U}^g$

Relationship: $\bar{U}^g = x\bar{U}_1^g + (1-x)\bar{U}_2^g$

$$\bar{U}_1^g = yz\Gamma_{11}^g + y(1-z)\Gamma_{12}^g + (1-y)z\Gamma_{21}^g + (1-y)(1-z)\Gamma_{22}^g$$

$$\bar{U}_2^g = yz\Gamma_{11}^g + y(1-z)\Gamma_{12}^g + (1-y)z\Gamma_{21}^g + (1-y)(1-z)\Gamma_{22}^g$$

$$\Gamma_{11}^g = \theta\left(T - e_s \cdot q - e_1^3\right) - \left(e_s \cdot q - e_1^3\right) - c_1$$

$$\Gamma_{12}^g = \theta\left(T - e_t \cdot q - e_2^3\right) - \left(e_t \cdot q - e_2^3\right) - c_1$$

$$\Gamma_{21}^g = \theta\left(T - e_1^2 \cdot q - e_1^3\right) - \left(e_1^2 \cdot q - e_1^3\right) - c_1$$

$$\Gamma_{22}^g = \theta\left(T - e_1^2 \cdot q - e_2^3\right) - \left(e_1^2 \cdot q - e_2^3\right) - c_1$$

$$e_s = \frac{k_2(\lambda + bh)(a - bc - bhe_0)}{2k_1(2bk_2 - u^2) - k_2(\lambda + bh)^2}, e_t = \frac{(\lambda + bh)(a - bc - bhe_0)}{4bk_1 - (\lambda + bh)^2}$$

$$h = A_1 x + A_2$$

Parameter changes: the probability that the producer adopts upgraded technology Y_1 is $y(0 \leq y \leq 1)$, and the probability that the retailer adopts low-carbon marketing strategy Z_1 is $z(0 \leq y \leq 1)$.

Significance: to find out under what circumstances should the government adopt the tolerant policy, and under what circumstances should it not.

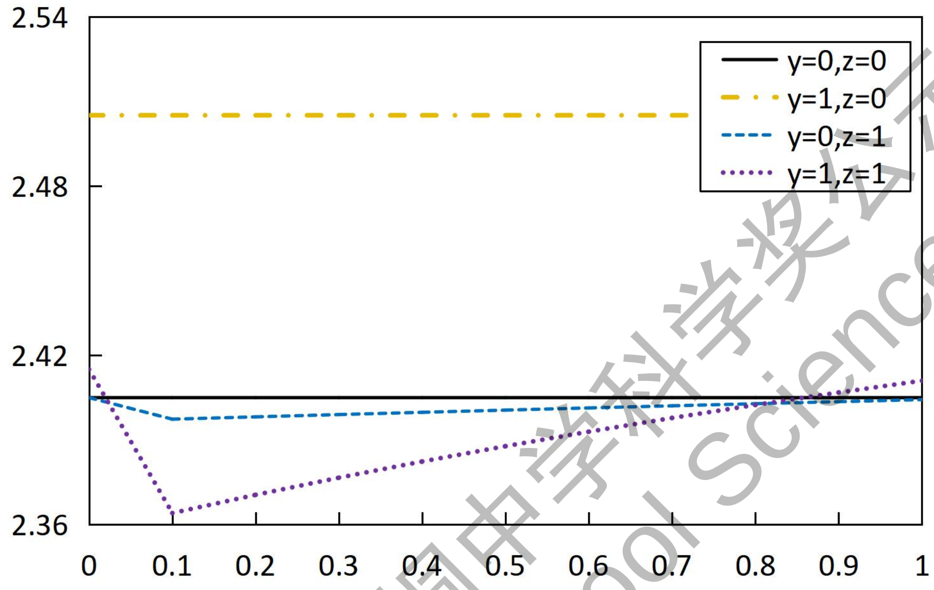

Figure 5.1. The government's expected payoff

As demonstrated in Figure 5.1 above that, by comparing horizontally, when the retailer chooses not to adopt low-carbon marketing strategies, regardless of the producer's decision to chooses upgrading the technology or not, the government obtains the same payoff, no matter whether it adopts tolerant or strict strategies. When the retailer chooses low-carbon marketing and the producer chooses not to upgrade the technology, the government can obtain greater government payoff by adopting the strict strategy than adopting the tolerant strategy. When the retailer chooses low-carbon marketing and the producer chooses to upgrade the technology, the government can also obtain greater government payoff by adopting the strict strategy than adopting the tolerant strategy.

By comparing the results vertically, when the retailer chooses not to conduct low-carbon marketing, the producer can obtain the maximum payoff when it chooses to upgrade the technology.

5.2 The producer's expected payoff and the probability of upgrading technology

As the supplier of low-carbon emission products, the producer plays a mediating role between low-carbon productive consumption and non-productive consumption, and communicate the government's policy orientation to the retailer and consumer behavior. Innovation is the key for the producer to carry out low-carbon production strategy. In the long run, reducing energy consumption and improving production efficiency helps the producer reach an economic scale and significantly reduce production costs, bringing a positive economic payoffs. However, in the short term, investing in the technology increases costs and risks with lower investment returns. Based on this, we assume that:

Abscissa: the probability y of upgrading the technology

Ordinate: the producer's expected payoff $\bar{U}^s$

Relationship: $\bar{U}^s = y\bar{U}_1^s + (1-y)\bar{U}_2^s$

$$\bar{U}_1^s = z\Gamma_{11}^s + (1-z)\Gamma_{12}^s + h(xE_1 + (1-x)E_2)$$

$$\bar{U}_2^s = z\Gamma_{21}^s + (1-z)\Gamma_{22}^s + h(xE_1 + (1-x)E_2)$$

$$\Gamma_{11}^s = \frac{k_1k_2(a-bc-bhe_1^2)^2}{2[2k_1(2bk_2-u^2)-k_2(\lambda+bh)^2]}$$

$$\Gamma_{12}^s = \frac{k_1(a-bc-bhe_1^2)^2}{2[4bk_1-(\lambda+bh)^2]}$$

$$\Gamma_{21}^s = \frac{k_2(a-bc-bhe_1^2)^2}{4(2bk_2-u^2)}$$

$$\Gamma_{22}^s = \frac{(a-bc-bhe_1^2)^2}{8b}$$

$$e_s = \frac{k_2(\lambda+bh)(a-bc-bhe_0)}{2k_1(2bk_2-u^2)-k_2(\lambda+bh)^2},\ e_t = \frac{(\lambda+bh)(a-bc-bhe_0)}{4bk_1-(\lambda+bh)^2}$$

Parameter changes: the probability that the government adopts the tolerant strategy X_1 is $x(0 \le x \le 1)$, and the probability that the retailer adopts a low-carbon marketing strategy Z_1 is $z(0 \le z \le 1)$

Significance: to find out when the producer should choose to upgrade the technology and when it should not to.

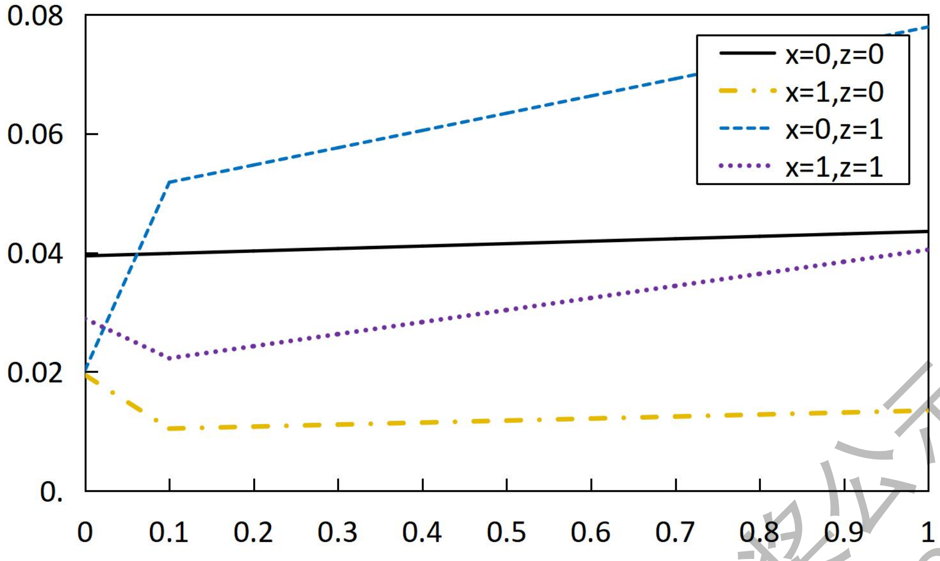

Figure 5.2. The producer's expected payoff

As demonstrated in Figure 5.2 above, by comparing horizontally, when the government adopts tolerant policy and the retailer chooses low-carbon marketing, the producer may choose to upgrade technology to obtain the maximum expected payoff. When the government adopts tolerant policies and the retailer chooses not to low-carbon marketing, the producer should choose not to upgrade the technology in order to obtain maximum expected payoff. When the government adopts strict policy and the retailer chooses low-carbon marketing, the producer should choose the upgraded technology to significantly improve its expected payoff. When the government adopts strict policy and the retailer chooses not low-carbon marketing, the producer should still choose the upgraded technology to obtain the maximum expected payoff, but the increase is relatively small.

Comparing the results of all strategies vertically, when the government adopts strict policy and the retailer chooses low-carbon marketing, the producer should choose to upgrade the technology to obtain the maximum expected payoff.

5.3 The retailer's expected payoff and the probability of low-carbon marketing

With the improvement of their knowledge level and living standard, consumers have gradually fostered the awareness for environmental protection and are becoming increasingly willing to buy green, low-carbon products. However, for consumers, the commodity price is still the key consideration factor when purchasing, and rational consumers in the economic sense are more willing to choose products with lower prices under the same conditions. In most cases, consumers are not the direct beneficiaries of government green subsidies, and the retailer plays an important role in distributing the incentives. When the government distributes green subsidies through retailers, retailers may either transfer the green subsidies to consumers through low-carbon marketing, or they can choose not to market in a low-carbon way and take the consequences. Based on this, we assume that:

Abscissa: probability z of low-carbon marketing

Ordinate: producer's expected payoff $\bar{U}^r$

Relationship: $\bar{U}^r = z\bar{U}_1^r + (1-z)\bar{U}_2^r$

$$\bar{U}_1^r = y\Gamma_{11}^r + (1-y)\Gamma_{21}^r$$

$$\bar{U}_2^r = y\Gamma_{12}^r + (1-y)\Gamma_{22}^r$$

$$\Gamma_{11}^r = \frac{k_1^2 k_2(2bk_2 - u^2)(a - bc - bhe_1^2)^2}{2[2k_1(2bk_2 - u^2) - k_2(\lambda + bh)^2]^2}$$

$$\Gamma_{12}^r = \frac{bk_1^2(a - bc - bhe_1^2)^2}{[4bk_1 - (\lambda + bh)^2]^2}$$

$$\Gamma_{21}^r = \frac{k_2(a - bc - bhe_1^2)^2}{8(2bk_2 - u^2)}$$

$$\Gamma_{22}^r = \frac{(a - bc - bhe_1^2)^2}{16b}$$

$$e_s = \frac{k_2(\lambda + bh)(a - bc - bhe_0)}{2k_1(2bk_2 - u^2) - k_2(\lambda + bh)^2}, e_t = \frac{(\lambda + bh)(a - bc - bhe_0)}{4bk_1 - (\lambda + bh)^2}$$

$$h = A_1 x + A_2$$

Parameter change: the probability that the government adopts the tolerant policy X_1 is $x(0 \le x \le 1)$, and the probability that the producer adopts the upgraded technology Y_1 is $y(0 \le y \le 1)$.

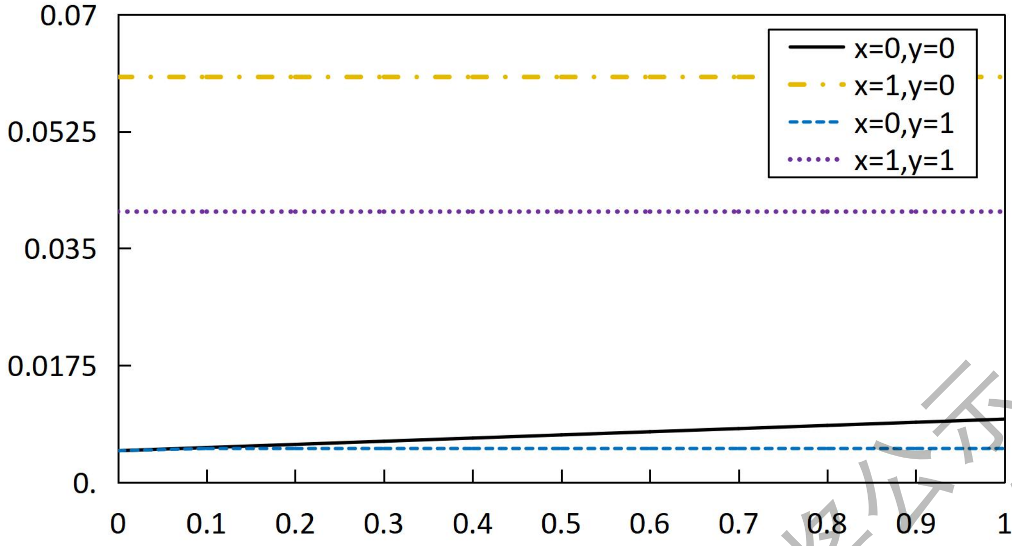

Figure 5.3. The retailer's expected payoff

As demonstrated in Figure 5.3 above, by comparing horizontally, when the government adopts tolerant policy, the expected payoff of the retailer is constant, no matter whether the producer chooses to upgrade technology or not and no matter whether the retailer chooses low carbon marketing or not. When the government chooses strict policy and the producer chooses to upgrade technology, the retailer's expected payoff remains constant, no matter whether the retailer chooses low carbon marketing or not. When the government chooses strict policy and when the producer chooses not to upgrade the technology, the retailer can obtain the maximum expected payoff by choosing low-carbon marketing.

Comparing the results of all strategies vertically, when the government chooses the tolerant policy and the producer chooses not to upgrade the technology, the retailer can obtain the maximum expected payoff.

6. Conclusions

Reducing carbon emission has become a critical measure to address global warming challenges. Low-carbon supply chain plays important role to achieve governments' carbon emission reduction policy objectives. Existing researches mainly focus on game analysis of two parties, either between the government and the producer, or between upstream and downstream of the supply chain. However, as government plays a leading role by setting carbon emission target, and both the producer and the retailer are key players in the low-carbon supply chain, a tripartite game analysis is meaningful to better understand the real world evolutionary dynamics.

In this paper, we conducted the tripartite game analysis among the government, the producer, and the retailer. The game model is applied to reach the equilibrium point among three parties. Then the Evolutionary game theory is used to achieve the evolutionary stable strategy. Lastly, we conducted a Matlab simulation study to demonstrate the tripartite evolutionary equilibrium model.

The results show that each of government's carbon emission strategy generates its corresponding equilibrium point, and the strict carbon emission strategy may lead the producer to adopt advanced technologies to reduce carbon emission. In addition, the stability of the equilibrium point among the government, the producer and the retailer will not be affected by whether the government implements tolerant or strict policy. The sensitivity coefficient of consumers to carbon emission and the sensitivity coefficient of consumers to marketing effort affect the stability of the equilibrium point of the system evolution.

As a policy advice, because the government's carbon emission policy will lead the supply chain to respond accordingly, it is clear that the government plays a vital role in achieving low carbon emission objectives and the government should promote such policy whenever possible after taking overall considerations on social payoffs. Raising public awareness through education and economic incentives will greatly improve the policy effectiveness through impacting the retailer's and producer's decision.

Appreciation Note:

It has been my long time desire to contribute to environmental protection. For me, living in a harmonious community with a clean and sustainable environment should be the fundamental pursuit of everyone. A severe and acute pneumonia sent me to ICU when I was 5 years old, and through that experience I realized that since the air pollution was one of the reasons that led to the disease infecting many people, it should be paid more attention to. Since recovering, I have been participating in multiple environmental protection activities, volunteering in various forms of communities services, and publishing articles to promote public awareness. As I grow up, I learned that there are many policies and economic tools already implemented to alleviate the problem, yet more efforts from all aspects of the society may be needed to broader the impacts to protect the environment. The S.-T.Yau High School Science Award provides me with an unique opportunity to explore the possibilities and put into practice what I have learned from school to resolve critical issues.

My appreciation goes to Dr. Jingchun SU. Dr. Su is my advisor, and she guides me for free and understands well on my passion in both the academic studying and environmental protection. When we discussed the selection of research topics, she often encouraged me to be confident to think big, and to think creatively. Dr. Su also advised me to apply what I have learned in high school and through extra curricular activities to understand and improve the existing model, especially the calculus and probability theory. Special thanks also goes to Jin FEI and Chaoyang CHENG, both have given me great advice and supports in the modeling process as well as conducting the simulation work.

It has been a great learning experience. I know that this paper is far from good, and there are lots of room for improvement. I will continue to build my skills and knowledge in financial modeling and game theory study, and to further expand the research around this paper's topic. I will also continue to follow up the latest development in the green policy in various industries, and try to perform the real life case study as more data become available.

References

Bellassen V., Stephan N., Afriat M., Alberola E., Barker A, & Chang J.P.(2015). Monitoring, reporting and verifying emissions in the climate economy. Nature Climate Change 2015;5(4):319-28.

Diao X.W., Zeng Z.X., & Sun C. (2021). Collaborative research on supply chain of two products under mixed carbon policy. Chinese Journal of Management Science, 2021.29 (02) : 149-159.

Friedman, D. (1991). Evolutionary games in economics. Econometrica, 1991, 59 (3): 637-666

Geels F W, Sowacool B K, & Schwaanen T. (2017). Sociotechnical transitions for Deep decarbonization. Science. 2017.357 (6357): 1242.

Guo, D.Y., Chen, H., & Long, R.Y. (2018). The allocation of government for initial carbon allowance in downstream carbon trading market. China Population Resources and Environment, 2018, 28 (4): 43-54.

Jiang, J.J., Xie, D., Ye, B., Shen, B., & Chen, Z.M (2016). Research on China's cap-and-trade carbon emission trading scheme: Overview and outlook. Applied Energy 178, 902–917.

Jiang, S.Q. & Sun, Y.J. (2016). Game analysis on the initial allocation of carbon trading amount. F224.32; x196.

Kong, X.Z. (2013). Policy proposals on developing carbon emission trading scheme to dissolve the overcapacity in China's cement industry. China Cem, 8:12–4 (in Chinese).

Li, X.J., Chen, M.N., & Da, Q.L. (2020). Optimization decision of closed-loop supply chain of automobile manufacturing enterprises under government regulation from the perspective of carbon trading. Management Review, 2020.32 (05):269-279 (in Chinese).

Liao, N., Lu, C. & He, Y. (2021). Study on energy saving company's participation to the supply chain carbon emission reduction cooperation under carbon trading policy. Chinese Journal of Management Science, 2021.29 (02): 160-167 (in Chinese).

Luo, R.L., Fan, T.J., Li, S.X., & Li, X.P. (2014). The study of carbon emission permits allocation in China's petrochemical industry. China Soft Sci, 2:171–8 (in Chinese).

Hyon, J. & Kim J.O. (2017). Korea's approach to overcoming difficulties in adopting the emission trading scheme. Climate Policy 2017;17(8):947-61.

Segnon, M., Lux, T., & Gupta, R. (2017). Modeling and forecasting the volatility of carbon dioxide emission allowance prices: A review and comparison of modern volatility models. Renewable and Sustainable Energy Reviews 69, 692-704.

Tang L., Wang H., & Li L.(2020). Quantitative models in emission trading system research: A literature review. Renewable and Sustainable Energy Reviews 2020;132:110052.

Venmans F. (2012). A literature-based multi-criteria evaluation of the EU ETS. Renewable & Sustainable Energy Reviews 2012;16(8):5493-510.

Wang, W.L, & Cheng, T.Y. (2021). Evolution game analysis of supply chain operations decision under the background of carbon trading. Systems Engineering - Theory & Practice, 2021, Vol.41(5): 1272-1281. (in Chinese)

Wang X.P., Qie S.Y.(2020). Research on the CCS Investment Timing in the supply chain under carbon trading mechanism. Journal of management engineering, 2020,34 (02) : 124-130.

Xia L.J., Kong Q.Y., Li Y.D., & Xu C.Q. (2021). Research on emission reduction and pricing decision of low-carbon supply chain considering cross-shareholding. China Management Science, 2021.29 (04) : 70-81. (In Chinese)

Xian, Y.Y., De, Q.Z., Xiu, Z.S., & Chang, C.T. (2020). Integration of tradable green certificates trading and carbon emissions trading: How will Chinese power industry do? Journal of Cleaner Production.

Ye,Y.X., Zheng, X.J., & Lin. S.C. (2022). Exploring the design of China's carbon tax system under the carbon peaking and carbon neutrality targets. Trade Fair Economy, X196; F812.42.

Yuan, K.F, Wu, G.Q, He, Bo, Ren, T.H., & Wang, D.F. (2022). Supply chain pricing of the quanlity upgraded re-manufacturing under cargo trade. Computer Integrated Manufacture System, 1-29 (2022-03-16).

Zhang, G.T., Wang, G.Q, Zhao, X.Y., & Liu, Y. (2021). Research on the close circle supply chain production and carbon trading strategy under carbon quota trading system. Chinese Journal of Management Science, 2021, 29 (01): 97-108 (in Chinese).

Zhang, Z.M., Yang, L.J., & Feng, B. (2022). Research on equilibrium optimization of carbon trading market: based on evolutionary game perspective. Journal of Central South University of Forestry & Technology (social science), Vol.16 No.4.

Zhu, Y., Li, Y.P., & Huang, G.H. (2013). Planning carbon emission trading for Beijing's electric power systems under dual uncertainties. Renew Sustain Energy Rev., 23:113-28.

Zhu, Y., Li, Y.P., Huang, G.H., Fan, Y.R., & Nie, S. (2015). A dynamic model to optimize municipal electric power systems by considering carbon emission trading under uncertainty. Energy, 88:636-49.

Appendix I

Combination of behavior strategies, in details:

Behavior strategies	Government's Payoff	Producer's Payoff	Retailer's Payoff
(X_1, Y_1, Z_1)	$\theta(T - e_s \cdot q - e_1^3) - (e_s \cdot q - e_1^3) - c_1$	$\frac{k_1 k_2 (a - bc - bhe_1^2)^2}{2[2k_1(2bk_2 - u^2) - k_2(\lambda + bh}$ $+ hE_1$	$\frac{k_1^2 k_2 (2bk_2 - u^2)(a - bc - bhe_1^2)}{2[2k_1(2bk_2 - u^2) - k_2(\lambda + bh)^2]}$
(X_1, Y_1, Z_2)	$\theta(T - e_t \cdot q - e_2^3) - (e_1^2 \cdot q - e_2^3) - c_1$	$\frac{k_1 (a - bc - bhe_1^2)^2}{2[4bk_1 - (\lambda + bh)^2]} + hE_1$	$\frac{bk_1^2 (a - bc - bhe_1^2)^2}{[4bk_1 - (\lambda + bh)^2]^2}$
(X_1, Y_2, Z_1)	$\theta(T - e_1^2 \cdot q - e_1^3) - (e_1^2 \cdot q - e_1^3) - c_1$	$\frac{k_2 (a - bc - bhe_1^2)^2}{4(2bk_2 - u^2)} + hE_1$	$\frac{k_2 (a - bc - bhe_1^2)^2}{8(2bk_2 - u^2)}$
(X_1, Y_2, Z_2)	$\theta(T - e_1^2 \cdot q - e_2^3) - (e_1^2 \cdot q - e_2^3) - c_1$	$\frac{(a - bc - bhe_1^2)^2}{8b} + hE_1$	$\frac{(a - bc - bhe_1^2)^2}{16b}$
(X_2, Y_1, Z_1)	$\theta(T - e_s \cdot q - e_1^3) - (e_s \cdot q - e_1^3) - c_2$	$\frac{k_1 k_2 (a - bc - bhe_1^2)^2}{2[2k_1(2bk_2 - u^2) - k_2(\lambda + bh}$ $+ hE_2$	$\frac{k_1^2 k_2 (2bk_2 - u^2)(a - bc - bhe_1^2)}{2[2k_1(2bk_2 - u^2) - k_2(\lambda + bh)^2]}$
(X_2, Y_1, Z_2)	$\theta(T - e_t \cdot q - e_2^3) - (e_t \cdot q - e_2^3) - c_2$	$\frac{k_1 (a - bc - bhe_1^2)^2}{2[4bk_1 - (\lambda + bh)^2]} + hE_2$	$\frac{bk_1^2 (a - bc - bhe_1^2)^2}{[4bk_1 - (\lambda + bh)^2]^2}$
(X_2, Y_2, Z_1)	$\theta(T - e_2^2 \cdot q - e_1^3) - (e_2^2 \cdot q - e_1^3) - c_2$	$\frac{k_2 (a - bc - bhe_1^2)^2}{4(2bk_2 - u^2)} + hE_2$	$\frac{k_2 (a - bc - bhe_1^2)^2}{8(2bk_2 - u^2)}$
(X_2, Y_2, Z_2)	$\theta(T - e_2^2 \cdot q - e_2^3) - (e_2^2 \cdot q - e_2^3) - c_2$	$\frac{(a - bc - bhe_1^2)^2}{8b} + hE_2$	$\frac{(a - bc - bhe_1^2)^2}{16b}$