A Mathematical Framework of Interactions in the Metaverse

Author: Yinkai Liu

Region: Shanghai

Country of Origin: China

Mentor: Maximilien MacKie

A Mathematical Framework of Interactions in the Metaverse

Yinkai Liu

12th September 2024

Keywords: Evolutionary Game Theory, Markov Chains, Exponential Equation Modeling

Abstract

Virtual worlds such as META play important social and economic roles. With the development of virtual reality technology, future societies could have foresee transactions, have a separate identity, or even become employed in the simulated world. This paper does not concern individual behavior on the metaverse, but rather interactions between civilizations, including extraterrestrial ones, spanning a multitude of dimensions and technological capabilities. We adopt the lens of game theory, a common tool in strategic decision making, and exponential equation modeling to simulate realistic communication scenarios between civilizations. We will use exponential modeling to determine equations of cost and benefit of different types of interactions. We will also outline more complex theory including the radius of detection, truths and lies, and use Markov chains to model the probabilistic and casual transitions between events. Finally, we will use MATLAB simulations and numeric methods to analyze hypothetical situations regarding trade and dimensions.

Contents

1 Introduction

In the past few years, the global attention of people has been caught by the idea of virtual worlds especially with the emergence of META. These digital spaces serve significant social and economic functions, providing opportunities for engaging in advanced commerce and virtual identity, as well as employment within the virtual settings. As technology of virtual reality (VR) develops, it raises further concerns about how society will develop in the next generation, living in such environments and perhaps even exchanging with the capabilities of arbitrary experiences that are currently not considered possible in the world.

However, there are worries regarding the exact nature of such interactions taking place in a virtual world, especially in terms of trying to interact with advanced civilizations up to the extent of communicating and making joint efforts with aliens. This study focuses on civilizational communication through the use of game-theoretic approaches, the model of conflict and social processes, and mathematical simulation. We examine in greater detail more sophisticated models such as the radius of detection, truth vs deception in communication, and transitions between events in terms of probabilities and Markov chain models. This study provides insights into how inter-civilizational interactions, facilitated by advanced VR and communication technologies, might unfold in a future dominated by the virtual world.

1.1 A Brief History of Metaverse and Interactions

The concept of the metaverse, a virtual reality space where users interact with a computer-generated environment and each other, dates back to the early 1990s when Neal Stephenson first coined the term in his novel *Snow Crash*. The idea envisioned a future where digital avatars and immersive worlds would revolutionize social and economic interactions. As technology progressed, platforms like Second Life in the early 2000s provided a glimpse of what the metaverse could offer—virtual communities, digital economies, and new forms of social engagement. [10]
By the 2020s, with the advent of platforms like META, the metaverse became a tangible reality, supported by advancements in VR, blockchain, and decentralized technologies. These virtual worlds now host various interactions, including commercial transactions, social gatherings, and even education. The metaverse has become a thriving digital economy, where virtual property, NFTs, and cryptocurrencies play a pivotal role. At the same time, the idea of interactions with sentient beings—whether virtual or extraterrestrial—parallels the goals of SETI (Search for Extraterrestrial Intelligence). Like SETI's efforts to detect extraterrestrial life, the metaverse pushes the boundaries of how humans might engage with unknown entities, even those of different technological capacities or dimensions. As both fields evolve, they raise questions about how civilizations—whether human, virtual, or extraterrestrial—could interact, and what the strategic implications of these encounters might be.
Today, with rapid developments in metaverse technologies and the search for intelligent life through tools like the James Webb telescope, studying the strategies for engaging with unfamiliar civilizations—whether in the virtual world or beyond—becomes crucial, influencing the future of societal and economic structures.

1.2 Previous Mathematical Expositions in Interaction Modeling

In 1960, Frank Drake proposed a statistical model to estimate the number of alien civilizations in the galaxy.

$$N = G f_p n_e f_l f_i f_c$$

whose variable definitions can be found in the References section.
Many coefficients of the equations are difficult to measure or arbitrary, increasing the error of the results. The only variable that is known with any precision is G, the rate at which stars form, which is at about 7 per year. In addition, there is an assumption that life can only exist on a planet similar to Earth. We therefore do not use this model.

The Fermi Paradox is the discrepancy between the lack of conclusive evidence of advanced extraterrestrial life and the apparently high likelihood of its existence.

Indeed, merely in the Milky Way alone, there are billions of stars similar to the Sun; and with high probability, a large array of stars have planets in the habitable life zone. Many of the planets are much older than Earth, meaning that if intelligent life exists, they are highly likely to have developed by now. There are many potential explanations for the Fermi Paradox. For instance, intelligent extraterrestrial civilizations may be very rare or the lifetime of such civilizations are short. Further studies must be done to empirically investigate this issue.

There are several attempts in the literature to model communication between civilizations using game theory. This section will focus on [3]. In this sequential game, one civilization takes it turn first, and then the other civilization makes the decision. Additionally, each civilization has two strategies, to do nothing or annihilate one another. As illustrated, it is a dominant strategy to annihilate the other as soon as possible, since it ensures positive payoff while the other civilization is not able to annihilate the first.

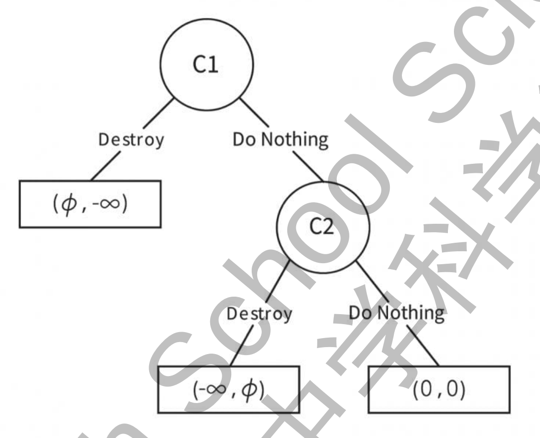

Figure 1: Decision Tree in SETI Interactions [3]

The model above does not justify the sequence of decision making. In addition, annihilation is not a sudden process and takes time, so the second civilization can still choose to start destroying the first once the first proclaims interstellar war. Finally, it only considers two methods of interaction: annihilation and doing nothing, which is somewhat limited.

Our paper encompasses a wide range of game theory prerequisite knowledge. To facilitate understanding, below is an introduction to game theory.
In *sequential games*, players' decisions unfold over time, opposite to simultaneous scenarios. The *Prisoner's Dilemma* exemplifies the tension between cooperation and defection, offering insights into how rational beings might choose suboptimal outcomes, a phenomenon potentially applicable to interstellar civilizations' strategy and credibility. *Nash Equilibrium*, pivotal in this context, delineates a state where no civilization can benefit by unilaterally changing its strategy. [1]
Information sharing games are also significant. Player knowledge is modeled through information partitions, dividing the set of all possible states, Ω, into subsets where each subset indicates indistinguishable outcomes for a player. When two players share information, their combined knowledge narrows down the possible true state to the intersection of their individual information partitions.[2]
These concepts collectively form a robust theoretical foundation for mapping optimal interactions between extraterrestrial entities. For more information, see book and Slikker's paper.

2 Nature of Metaverse Interactions

2.1 Potential Motivations for Contact

2.1.1 War and Combat

The prospect of war and combat as motivations for interstellar contact reflects a darker side of potential encounters, drawing from human history where advanced civilizations have exploited less advanced ones. This perspective suggests that an advanced alien civilization might view less advanced ones not as partners but as resources or obstacles, similar to historical human exploitation of other species and cultures.

2.1.2 Information Sharing

Information sharing is where civilizations exchange knowledge for mutual benefit. This cooperation could lead to significant advancements for both parties, but it also comes with risks, particularly if one civilization becomes too transparent, potentially exposing itself to vulnerabilities.

2.1.3 Commerce and Technology Advancements

Commerce as a motivation for interstellar contact is driven by the potential for civilizations to trade unique resources, offering economic benefits through specialization and exchange. However, the current impracticality of interstellar trade, due to the exorbitant costs of space travel, limits this motivation. Future advancements in technology could change this, making commerce a more viable reason for contact.

2.2 The Interactions Scale

All of the motivations mentioned above can be grouped into positive or negative valence. We can describe each type of interaction as a pair $(expl, exch)$, where q is a real number in the range of $[-1,0] \cup \{1\}$: when q is negative, the stronger civilization is involved in $(|q| \times 100)\%$ exploitation, in other words, the weaker civilization is deprived of the given ratio of population and resources. When $q = 1$, the two civilizations engage in complete information sharing. On the other hand, $exch$ is a real number in the range of $[0,1]$, representing that commerce between two civilizations can satisfy $(exch)\%$ of each civilization's material needs. Note that $exch$ can only take positive values when $q \geq 0$. The relationships between different types of interactions are shown in the figure below (for convenience, negative valence interactions are to the right, whereas positive valence interactions are to the right).

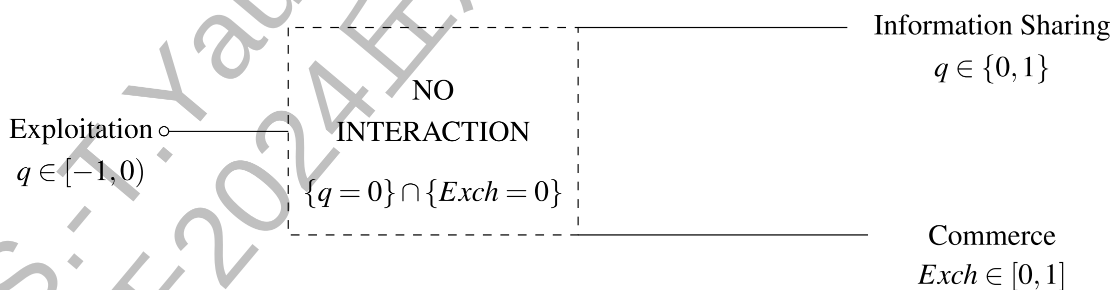

Figure 2: The Interactions Scale

3 Interaction Between Two Civilizations

3.1 Characteristics of Civilizations

See Table 1 for a comprehensive list of all characteristics that can affect interactions between civilizations.

Characteristic	Mathematical Representation	Explanation
Dimension	$d \in \{2,3,4,5\}$	In extraterrestrial intelligence, dimensions significantly limit the scope particular organisms perceive the world around them. For instance, 4D civilizations may enable faster-than-light travel and 5D civilizations may perceive parallel universes. Therefore, Higher-order civilizations could own significantly more information than us due to innate perspectives.
Kardashev Civilizations Scale	$K = \frac{log_{10}W-6}{10}$	The Kardashev scale defines a civilization's development of science and technology with an input of energy usage rate, since the ability to harness energy is strongly correlated with the extent to which a civilization is advanced. A Type 1 civilization can harness the energy of the entire planet, Type II can use the energy of the star, while Type III can harness the energy of the entire galaxy.
Distance between Civilizations	r, in standard astronomical units	Since we are considering Extraterrestrial communications, the cost of interstellar travel can be quite high. Once the coordinates are exposed, the distance between civilizations can directly impact the difficulty of both commerce and war. Specially, when two civilizations are located in different parallel universes or invisible to each other, the distance is defined as ∞
Natural resources	$h = \begin{pmatrix} a_{1,1},\cdots,a_{1,n} & b_{1,1},\cdots,b_{1,n} \\ \vdots & \vdots \\ a_{m,1},\cdots,a_{m,n} & b_{m,1},\cdots,b_{m,n} \end{pmatrix}$	This parameter decides the value of natural resources to the particular civilizations, which is essential to calculating the benefits of both commerce and control. $a_{i,j}$ represents the unit cost of resource j in civilization i in billion dollars, while $b_{i,j}$ represents the annual usage rate of resource j in civilization i in tonnes. Note that since not all civilizations are carbon-based, they may utilize different resources and thus not all may be taken into consideration.
Population	p, in millions	Population is a measurement of how large a civilization is, generally determining the amount of human resources a stronger civilization can utilize. It is also correlated with the cost of interstellar war.

Table 1: List of all Characteristics

3.2 Derivation of the Model

For all equations and calculations below, the subscript below each variable represent the civilization number.

3.2.1 Technological Advancements

The value of information sharing includes the cost of developing to the shared state by the poorer civilization independently todobecause it is given for free. In addition, it includes using new technology instead of old technology in the original development years.

The developmental costs of technology increases exponentially as technology gets more advanced according to the Moire's law. The Kardashev scale increases by 1 when energy consumption is multiplied by 10^{10}, which is exponential, so the instantaneous developmental cost to progress plotted against the Kardashev scale is also exponential
Pre-industrial civilizations have a Kardashev scale of 0.4 and current human civilizations have a Kardashev scale of 0.7 [11]. Since technology is developing at an increasingly rapid rate, energy consumption is increasing exponentially, which lead to Kardashev scale increasing linearly. Through calculation based on slopes, year 2000 has Kardashev scale 0.6737 and year 2019 has Kardashev scale 0.6954. According to the graph, the R&D expenditures in 2000 is approximately \$610 billion, and in 2019 it is around \$2043 billion. This gives rise to the following equation:

$$610 = a \times b^{0.6737}$$
$$2043 = a \times b^{0.6954}$$

Solving this:

$$a = 3.35 \times 10^{-14}$$
$$b = 1.37 \times 10^{24}$$

However, solving for the equation is not adequate. For the benefits of information sharing, we need to consider the costs annually to develop to the state of the stronger civilization, not merely a single point on the function. In mathematical terms,we need to find the indefinite integral of the curve:

$$\int 3.35 \times 10^{-14}(1.37 \times 10^{24})^x \, dx = 6.03 \times 10^{-16}(1.37 \times 10^{24})^x$$

Similarly, we need to repeat the process of acquiring the perspective of a higher dimension. Dimensions, by definition, follows an exponential scale. For instance, 2D civilizations are limited to singular planes, whereas 3D civilizations can view solids and stereoscopic objects, allowing accesses to significantly more information. Thus, the function that maps dimension to revenue is also exponential, denoted as $y = a \cdot b^x$.

To solve the function, we consider two special points: the third and fourth dimension. The third dimension is a resemblance of our own civilization, which have a Kardashev scale of 0.4 prior to the industrial revolution. This translates to a value of $2.72 \cdot 10^{-6}$ billion dollars, according to the previous equation. The fourth dimension civilization has a natural ability to travel at the speed of light. Recall the formula of kinetic energy $E = \frac{1}{2}mv^2$, where m is mass and v is the velocity. Assuming that a person has mass 100 kilograms, accelerating it to the speed of light will require $4.5 \cdot 10^{18}$ joules of energy. Suppose that usage of energy is distributed between 100 seconds, and time travel accounts for merely 1% of the civilization's total use of power. This suggests that the civilizations' power is $4.5 \cdot 10^{18}$ watts, translating to a Kardashev civilization of 1.265. Substituting the value into the last equation, this equates to a value of \$$2.05 \cdot 10^{15}$ billion dollars. Hence:

$$2.72 \times 10^{-6} = ab^3$$
$$2.05 \times 10^{15} = ab^4$$

Solving this set of equation yields:

$$a = 6.35 \times 10^{-69}$$
$$b = 7.54 \times 10^{20}$$

Likewise, we are required to find the indefinite integral of this function. The result is:

$$\int 6.53 \times 10^{-69}(7.54 \times 10^{20})^x \, dx = 1.32 \times 10^{-70}(7.54 \times 10^{20})^x$$

From the last two equations, we can determine the strength of a civilization with its dimension and Kardashev civilizations scale. Observe that the exponent of dimension is 20, while the exponent of Kardashev civilizations scale is 24. This suggests that dimension is about 0.8 times impactful as the civilizations scale. With this information, we define the strength of a civilization $s = 10^{8d+10K}$.

The notion of building cables and sending electromagnetic waves across the universe may seem energy-consuming, but the costs are minuscule compared with physical interstellar trips. There may also be alternative ways of
On the other hand, there are indeed cases where information sharing is unpractical, specifically when the dimension difference between two civilizations is too large. A 5D civilization have the prominent feature of being able to travel between parallel universes. whereas humans do not, which disables the two civilizations from being known to each other, and thus, information sharing. Hence, information sharing is limited to when the two civilizations have dimension difference at most 1. When information sharing is not impossible, the cost is denoted as $+\infty$.

3.2.2 Commerce

In the discussion of commerce revenues, we imply the exchange of natural resources. Recall the definition of the natural resources array, as stated in the aforementioned characteristics table. Particularly, when a civilization does not need resource p, for example, the value of $b_{1,p}$ will be 0; when a civilization does not own resource p, the value of $a_{1,p}$ will be $+\infty$.

A civilization can benefit from commerce when the resource unit price in another civilization is lower than the its price, and it needs a relatively large quantity of the resource. The total revenue is the price difference of each advantageous resource summed together. Thus, we have:

$$ben_1 = \sum_{i=1}^{n} b_{1,i} \cdot max(a_{1,i} - a_{2,i}, 0)$$

$$ben_2 = \sum_{i=1}^{n} b_{2,i} \cdot max(a_{2,i} - a_{1,i}, 0)$$

The majority cost of commerce is likely to emerge from interstellar transportation since product exchange requires physical contact. The cost of travelling varies directly with the distance and inversely with the stronger civilization's strength. Assuming that $s_1 > s_2$ for below, the cost for a single journey is denoted as $\frac{ar}{s_1}$. According to NASA estimates, each Apollo project cost about \$20 billion, which is \$10 billion a single way.[7] The Apollo project occurred during 1970, and from the previous logic, the Kardashev scale in 1970 is estimated at 0.6118. Thus, the Earth civilization's strength index at that time was $10^{(0.6118\times10+3\times8)} = 10^{30.118}$. The moon's average distance from Earth is about 0.0025 astronomical units.[9] Hence:

$$20 = \frac{a \times 0.0025}{10^{30.118}}$$

Solving this:

$$a = 1.04 \times 10^{34}$$

Note that the above calculations only account for a single trip, whilst in interactions like commerce, multiple trips are required throughout the year to ensure the successful delivery of products. A conventional single space trip can deliver 25 tonnes of product at most. The total amount of products exchanged is the sum of annual usage rate of the particular resource in the higher-priced civilization. Thus, at least

$$\lfloor \frac{\sum_{i=1}^{n} b_{1,i} \cdot (a_{1,i} > a_{2,i}) + b_{2,i} \cdot (a_{2,i} > a_{1,i})}{25} \rfloor$$

single trips are required annually.

3.2.3 Control

When a civilization is partially controlled, the stronger civilization can utilize a portion of its natural and human resources at no cost. Since these two benefits are mutually independent, we can calculate them separately.

The value of natural resources is the product of each resource's annual usage rate and unit price, taken as a sum. The unit price here refers to the minimum price in the two civilizations:

$$Nat = \sum_{i=1}^{n} b_{2,i} \cdot min(a_{1,i}, a_{2,i})$$

The value of human resources directly vary with the strength of the stronger civilization, as well as the number of controlled population, as in the form of $pop = as_1 p_2$. As of information provided by the U.S. Labor Bureau, the current average full-time annual salary is \$53,490; in other words, \$ 5.4×10^{-5} billion. Thus:

$$5.4 \times 10^{-5} = 10^{31} \times 10^{-6} a$$

$$a = 5.4 \times 10^{-30}$$

The costs of warfare varies inversely with the difference of strength between the two civilizations and the population of the stronger civilization. It varies also directly with the square of the defeated civilization's population, since the larger a civilization is, the more the resistance and more difficult to eliminate the civilization completely. This can be written in the form of $war = \frac{ap_2^2 s_2}{s_1 p_1}$.

We use the example of Japanese-Chinese conflict during World War II to estimate the value of a.Japan population was 72 million at that time, while Chinese mortality due to military reasons totaled 25 million. As a conflict between post-industrial and pre-industrial nations, the Japanese's strength is approximately 100 times stronger, as measured in the 0.2 difference in Kardashev scale. Japan GDP has an average of \$250 billion. [4] Since most fascist nations spend $\frac{1}{3}$ of their GDP on military conflict, both directly and indirectly, the Japanese budget is around \$83 billion. [6]

We can proceed to solve the equation as follows:

$$83 = \frac{a \times (25)^2}{100 \times 72}$$

$$a = 956.2$$

Nonetheless, the equation above does not measure the true cost of interstellar warfare. Civilizations in space may not be carbon-based, or even utilize vastly different resources than in Earth. For instance, if a civilization is hydrophobic, it's strength is likely to be significantly weakened when engaging in war on Earth because it will die once touching the vast oceans on the planet. Likewise, Earth is likely to be more vulnerable to the foreign weapons because they are never seen before and humans are unprepared to directly face them.

To remedy this problem, we need to clarify which resources each civilization use and find the intersection between them. Define:

$$R_1 = \{i | b_{1,i} \neq 0\}$$
$$R_2 = \{i | b_{2,i} \neq 0\}$$
$$RI = |R_1 \cap R_2|$$

Since the number of similar resources between two civilizations follow a normal distribution, this is also true for the reduction of warfare cost it incurs. The formula of a normal distribution is:

$$f(x) = \frac{1}{\omega\sqrt{2\pi}} exp(-\frac{(x-u)^2}{2\omega^2})$$

where μ is the mean and ω^2 is the variance of the normal distribution, and $exp(x)$ is the mathematical abbreviation for e^x.

We will limit the normal distribution function's domain to the strictly increasing portion. As at most 10 resources are included in the scope of this paper, the domain for normal distribution is $[0,10]$. When the two civilizations have 0 similarities, the cost is likely to be reduced by $\frac{1}{2^{10}} = \frac{1}{1024}$. Using the normal distribution calculator, parameters $\mu = 10$ and $\omega = 3$ most closely fits with the model.

3.2.4 Summary of Equations

Formally, given an interaction with parameters $(q, exch)$, assuming that $s_1 > s_2$, the net revenues are defined as the following:

$$\begin{aligned} Revenue_1 = {} & \sqrt{max(q,0)} \cdot \mathbb{I}_{K_1<K_2} \cdot \int_{K_1}^{K_2} 3.35 \times 10^{-14} (1.37 \times 10^{24})^x \, dx + \mathbb{I}_{d_1<d_2} \cdot \\ & \int_{d_1}^{d_2} 6.53 \times 10^{-69} (7.54 \times 10^{20})^x \, dx) + exch \cdot \sum_{i=1}^{n} b_{1,i} \cdot max(a_{1,i} - a_{2,i}, 0) + \\ & + \mathbb{I}_{q<0} \cdot (-q) \cdot (\sum_{i=1}^{n} b_{2,i} \cdot min(a_{1,i}, a_{2,i}) + 5.4 \times 10^{-30} s_1 p_2) - \mathbb{I}_{|d_1-d_2|>1} \cdot \infty \\ & - exch \cdot \lfloor \frac{\sum_{i=1}^{n} b_{1,i} \cdot \mathbb{I}_{a_{1,i}>a_{2,i}} + b_{2,i} \cdot \mathbb{I}_{a_{2,i}>a_{1,i}}}{25} \rfloor \cdot \frac{1.04 \times 10^{34} r}{s_1} - \mathbb{I}_{q<0} \\ & \cdot (-q) \cdot \frac{956.2 p_2^2 s_2}{s_1 p_1} \cdot \frac{1}{3\sqrt{2\pi}} exp(-\frac{(x-10)^2}{18}) \end{aligned}$$

where $\mathbb{I}_{a<b}$ is the indicator function for $a < b$.
The equation for the revenue of civilization 2 is defined in a similar manner.

When $q > 0$, in other words, the two civilizations are in a positive valence interaction, $revenue_{net} = min(revenue_1, revenue_2)$. This is because civilizations will only cooperate when the action is mutually beneficial.

When $q < 0$, or when two civilizations exploit or attempt to control each other, $revenue_{net} = max(revenue_1, revenue_2)$. In interstellar war, one civilization will always benefit at the cost of another, so how much this civilization benefits is of the most concern in this scenario.

4 Information Sharing in the Metaverse

In the case of interactions, not all characteristics of a civilization may be immediately apparent. The following are the three levels of information that could be known about a civilization:

- **Only the position of a civilization**. This occurs immediately after civilization detection.
- **Population and Kardashev Scale**. Known after any interaction has taken place.
- **Details enabling exploitation of the other civilization**. This includes specific military personnel, equipment, and passwords. Information like this is revealed only after collaboration to defeat a common enemy or when the civilization is infiltrated.

5 The Factor of Time

5.1 Radius of Detection

In the cosmos, civilizations are not initially known to each other. Let the dynamic radius of detection for civilization i be D_i. For civilizations a and b, they will only be exposed to each other when

$$D_a + D_b \geq r_{a,b}$$

The array D has two properties. Firstly, it is irreversible: there is no use lowering the radius of detection after being discovered by a stronger civilization, since the coordinates are already known. Moreover, it is solely determined by the civilization itself.

We will assume that technology places no upper limit on the radius of detection. Current human civilization could already detect all the observable universe in three dimensions. Cases of no detection could result in civilization being in different dimensions, which was discussed in the "Dimensions" section. Moreover there is no lower limit, since a civilization could completely insulate its signals.

5.2 Change in Detection Radius

Once per designated unit time, a civilization would reconsider the detection radius. The transition between detection radii can be modeled using an inhomogeneous Markov chain, since the current net benefit and radius causually determine the probability of the next change.

Markov chains are useful for modeling transition between different states in a stochastic (probabilistic) manner. A key property of Markov chains is that they are "memoryless," meaning the next state of the system depends only on the current state and not on the sequence of states that preceded it.

A Markov chain is defined by:
- A set of states $S = \{s_1, s_2, \ldots, s_n\}$.
- A transition matrix P, where each element P_{ij} represents the probability of transitioning from state s_i to state s_j.

Mathematically, we write:

$$P_{ij} = \mathbb{P}(X_{t+1} = s_j \mid X_t = s_i)$$

where X_t is the state of the system at time t, and P must satisfy:

$$\sum_{j=1}^{n} P_{ij} = 1 \quad \text{for all } i.$$

This ensures that the total probability of transitioning from any state s_i to all other states sums to 1. [5]

To determine the coefficients on the transition matrix, we consider the uncertainty involved. If a civilization increases radius of detection, it will have more risks being detected by an undesirable, stronger civilization which may be hostile according to the Dark Forest Hypothesis. Thus, there would be an exponentially decreasing function based on the extent of increase.

We define that

$$f(i,j) = \frac{1}{e^{j-i}}$$

Moreover, depending on the current net benefit, civilizations with lower net benefit will most likely view cooperation revenue as more promising than the risks, while those in the current state already has high net benefit would prefer to stay the same. This is relative to the civilization strength. Thus, we have

$$c = \frac{-qB}{10^K}$$

where B is the current net benefit, K is the civilization's Kardashev scale, and q is a constant.

We use this value to modify the previous exponential formula such that

$$g(i,j) = \frac{1}{e^{c(j-i)}}$$

This ensures for smaller net benefits, the exponential decrease is less steep, reflecting higher probability of radius increases.
Finally, we need to ensure all probabilities in a row add to be 1. Let

$$Y(i) = \int_{j=i}^{\infty} g(i,j)$$

After adjustment,

$$trans(i,j) = \frac{g(i,j)}{Y(i)}$$

Occasionally, changes in detection radius may occur irregularly due to unexpected reasons. For instance, when a Allie civilization reveals another civilization willing to cooperate, detection radius may be increased immediately. However, for the purpose of this paper, we ignore such cases.

5.3 Sequence of Decision Making

We can view the relationships between interstellar civilizations as a graph, and each pair of two civilizations make decisions accordingly about diplomacy. A civilization can take advantage to first make decisions about the interaction parameter when it first perceives the existence of another civilization. Upon the change in detection radius, all civilization pairs are scanned and those meeting (5.1) will undergo the decision process that maximizes (3.2.4).

6 Interaction Among Multiple Civilizations

The section oversees the further development of our general model concerning interaction between two civilizations, as outlined in the previous section. First, we will introduce the dark forest hypothesis and address some complications in interaction among larger civilization groups. Then, we will design experimental scenarios to explore the behavior of civilizations and their implications.

6.1 The Dark Forest Hypothesis

The dark forest hypothesis is a solution to the Fermi paradox, which asks why we have not yet found evidence of extraterrestrial life, despite the high probability that it exists. The hypothesis was first proposed in Liu Cixin's science fiction novel The Three Body Problem (2006), and it has since been taken seriously by some scientists and philosophers. [8]

The main concepts of the Dark Forest Hypothesis can be outlined as the following:

- The universe is a vast and dangerous place.
- There are many alien civilizations in the universe.
- Some of these civilizations are hostile and would not hesitate to destroy any other civilization that they perceive as a threat.
- The only way for a civilization to survive is to remain hidden and silent.

Under the dark forest hypothesis, any extraterrestrial civilization that has the capacity to traverse interstellar distances will promptly conceal itself. They will abstain from sending out any signals that could expose their whereabouts, and they will be constantly vigilant for other civilizations that they can annihilate before they are annihilated themselves. This hypothesis has a few implications for the communication of SETI (the search for extraterrestrial intelligence) civilizations. For instance, it implies that SETI civilizations should be cautious when transmitting signals that could disclose their location. It also implies that SETI civilizations should be cautious when interpreting any signals that they receive from other civilizations.

While the dark forest hypothesis is a powerful conceptual analysis of the dangers of interstellar communication, it has some limitations. For instance, it is only conceptual and lack numerical analysis to substantiate its claims. In addition, it assumes that all civilizations are hostile and fails to take into consideration the impact of a civilization's dimension and technological capability when it comes to interaction type.

In the next two section, we will strive to verify the Dark Forest Hypothesis through numerical computations. We will be utilizing the programming language MATLAB, optimal for calculation of large numbers and visualization. If the Hypothesis is broadly true, we will discuss potential refinements to the statement; otherwise, we will provide cases that contradict the hypothesis and formulate our unique conclusion.

6.2 Prerequisite to Simulations

6.2.1 Merging Civilizations

When one civilization conquers another civilization, some of the resources of the weaker civilizations will merge into the stronger civilization. This phenomena is illustrated in the following equations:

- $p^{'} = p_1 + (-q)p_2$
- $d^{'} = max(d_1, d_2)$
- $K^{'} = max(K_1, K_2)$ (Although the weaker civilization may add to the Kardashev scale, the scale is exponential with base 10, so even when the numerical value is quite large, the value will be extremely inconspicuous on a logarithmic scale.)
- $h^{'} = [(a_{1,1} \times \frac{b_{1,1}}{b_{1,1}+(-q)b_{2,1}}, \cdots, a_{1,n} \times \frac{b_{1,n}}{b_{1,n}+(-q)b_{2,n}}), (b_{1,1} + (-q)b_{2,1}, \cdots, b_{1,n} + (-q)b_{2,n})]$ (The total cost of producing a certain resource stays the same when a civilization engages in war to seize other civilization's resources. As a result, price level falls because there is more resources available).

Accordingly, weaker civilizations loose their resources when they are seized by the stronger civilizations. They gradually loses their population until it reaches 0: at this point, the civilization disappears from the universe.

Civilization	Dimension	Kardashev	Population	Position Coordinates
1	3	0.6	20	(2,2)
2	3	0.59	90	(2,3)
3	3	0.61	100	(3,2)
4	3	0.65	1	(0,0)
5	3	0.66	1	(5,5)

Table 2: Simulation 1 (Same Dimension)

6.2.2 Complications of Exploiting another Civilization

When a stronger civilization exploits a weaker civilization, one may be tempted to consider only the costs of war between these two civilizations. However, the truth is that the weaker civilizations may have cooperated with others before this decisions, and they may help resist the strong civilization's takeover. Therefore, we need to consider each of the weaker civilization's allies and their behavior, which can be classified into the following two cases:

- The ally is stronger than the civilization trying to conquer its cooperation partner ($cap_{temp} > cap_1$);
- The ally is not as strong as the civilization trying to conquer its cooperation partner ($cap_{temp} \leq cap_1$).

For convenience, denote the weaker civilization as civilization (a), the one that is trying to conquer the weaker civilization as civilization (b), and the weaker civilization's ally as civilization (c).

In the first case, civilization (c) can conquer civilization (b), betray civilization (a) and cooperate with civilization (b), or do nothing. If civilization (c)'s net benefit with civilization (a) exceeds both war cost and cooperation with civilization (b), civilization (c) will conquer civilization (b). In this scenario, civilization (b)'s net benefit of participating in exploitation is $-\infty$. When civilization (c)'s net benefit with (b) exceeds both net benefit with (a) and war cost, civilization (c) will betray (a). If this is true, civilization (b) can get the benefit of information sharing and commerce, but have the liability to conquer civilization (a) and other allies first.

Conversely, civilization (c) can choose to either cooperate with civilization (a) or wait for extinction in the second case. Optimally, if civilization (a)'s net benefit is greater than 0, civilization (c) will choose the first option. Otherwise, civilization (a) have to conquer (c) first to prevent further resistance.

Notice that there are many temporary relations discussed in the sections above. In MATLAB simulations, we will use the matrix *temrelat* to temporarily record them, and copy them to matrix *relations* if exploitation became the optimal interaction.

6.3 Simulation Experiments

In this section, we will conduct various experiments on some interesting properties of interstellar interactions and briefly explore some implications. We will use transparent circles with differing size to represent civilizations of distinct populations in the final outcome, where red circles represent isolated civilizations and every other color represent blocs of civilizations cooperating together. Sometimes, circles can overlap; however, that does not imply any kind of special relationship between civilizations: they are simply enlarged to make any changes more conspicuous. We also assume that the triangle inequality for planer distances is satisfied for the risk of violating empirical physical laws.

6.3.1 Interactions in the Same Dimension

Under most circumstances, interactions occur in communities where each civilization has the same dimension and technological capabilities are generally similar.

The community described in Table 4 consists of 3 ”developing” civilizations, with Kardashev scales ranging from 0.59 to 0.61, along with 2 relatively ”developed” civilizations, with Kardashev scales above 0.65. The less developed civilizations are clustered in the middle and have significantly more population than the more developed civilizations. This simulation is similar to the positions of major countries on Earth, with nations in Asia, Europe and the Americas. Results are shown below:

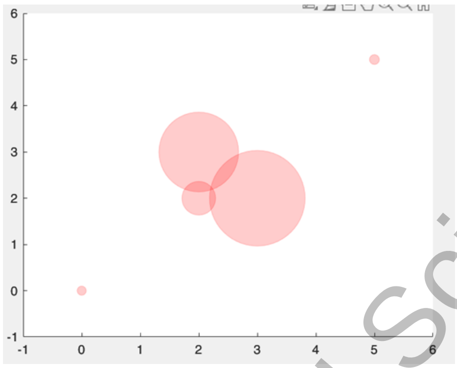

Figure 3: Results 1.1

At first, the 3 less developed civilizations showed a reluctance to cooperate. This is potentially because these civilizations have the same dimension, but distinct Kardashev values, so under no circumstance will information sharing lead to mutual benefit. Instead, the civilizations sneaked on each others, finding opportunity to control others’ resources, as Civilization 3 controlled 27% of Civilization 1 and 11% of civilization 2.

It is also surprising that the developed civilizations did not choose to control the less developed civilizations. A possible explanation lies in the significant difference in population. Neither did the two stronger civilizations cooperate with each other and form a bloc because of the large distance between them.
To explore variations of this simulation, we will change the population of Civilization 3 to 300 and leave all other characteristics the same. Please see figure 5 for the results of this round.

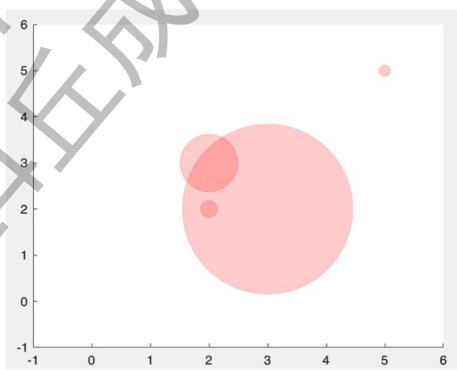

Figure 4: Results 1.2

This time, the increase in population of Civilization 3 made it more advantageous over the other two less developed civilizations, encouraging it to control more of their resources (80% and 53%) respectively. However, these actions also increased Civilization 3’s actions so that its Kardashev scale rises just above that of Civilization 4, making it destroy the latter civilization completely. This illustrates that in communities with similar capabilities, not necessarily the strongest civilization will take over the weakest. Instead, if a relatively weak civilization have advantages in population and location, it can easily merge in resources from others and change the interstellar dynamic.

Civilization	Dimension	Kardashev	Population	Position Coordinates
1	3	2	0.04	(1,1)
2	4	1.22	10	(3,1)
3	4	1.21	1	(2,4)

Table 3: Simulation 2 (Different Dimensions)

6.3.2 How Civilizations in Different Dimensions can Cooperate

In other cases, especially when a civilization decides to expose its coordinates to a relatively wider range, it may suddenly encounter civilizations in a different dimension. Table 5 and Figure 6 models and explains the results of a potential scenario.

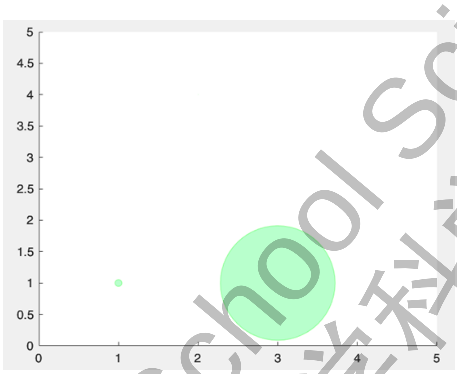

Figure 5: Results 2.1

In the scenario above, civilization 2 did not attempt to control civilization 1 despite the fact that they are in different dimensions. Primarily, the two civilizations had extremely similar capability values (a difference of merely 0.2), which increased the cost of war. Moreover, the human resources that civilization 1 had is comparatively low to the benefit of information sharing, since Civilization 1 can benefit from the higher dimension and Civilization 2 can benefit from the increased Kardashev level. Thus, the two civilization decides to cooperate.

When Civilization 3 is exposed to the system, it first makes a decision about its interaction type with Civilization 1. However, since Civilization 2 are allies with Civilization 1, and the cost of war with Civilization 3 (approx. 0.15 billion dollars) is significantly lower than the benefit with Civilization 1 (over 10^{13} billion dollars), Civilization 3 is successfully prevented from attacking Civilization 1. Later, however, when Civilization 3 meets Civilization 2, Civilization 2 destroys 3, which suggests that under some circumstances, higher dimension civilizations may attack each other instead and even help lower dimension civilizations.

Curiously, this result is only true when the conditions for Civilization 2 is unaltered. Even when the Kardashev scale of Civilization 2 increases up to 1.23, it tends to destroy both Civilizations 1 and 3, making it the sole civilization left in this community.

We can also explore variations to this simulation when the population of Civilization 1 varies. The correlation between population and the interaction type is shown in Table 6.

From the data in Table 6, when Civilization 1's population initially rises above the initial value 0.04 million, it is immediately antagonized by Civilization 2 due to the abundance of resources possible achieved by war. Only when Civilization 1's population approaches $6 \cdot 10^{13}$ million , it becomes less controlled by the other civilizations. The relations gradually change until Civilization 1's population reaches around 10^{15} million, when both higher-order civilizations are willing to cooperate with it.

Civilization 1 Population	Relations between 1 and 2	Relations between 1 and 3
0.04	1(Cooperation)	Initially cooperation, but 3 was destroyed by 2.
0.05	-1(Complete Control)	NaN
$6 \cdot 10^{13}$	-0.89	-0.51
10^{14}	-0.53	-0.07
$4 \cdot 10^{14}$	-0.13	No Interaction
10^{15}	1(Cooperation)	1(Cooperation)

Table 4: Results 2.2

The minimum population bar for cooperation is quite high, especially under current civilization sizes. This suggests that the efficiency of utilizing human resources is quite high for 4 dimensional civilizations or above. When we tested this scenario instead between $2D$ and $3D$ civilizations, the population bar becomes exponentially lower, enabling it to be possible to achieve under current circumstances.

6.3.3 A Treatise on Resources and Trade

Sometimes, trading can have a significant impact on interstellar interactions. See an example of a community that has not participated in trading in Table 7 and Figure 7.

Civilization	Dimension	Kardashev	Population	Position Coordinates
1	3	1	10^{-6}	(1,1)
2	3	2	10^{-6}	(2,1)
3	4	2	10^{-6}	(1.5,1.87)

Table 5: Simulations 3 (Trade)

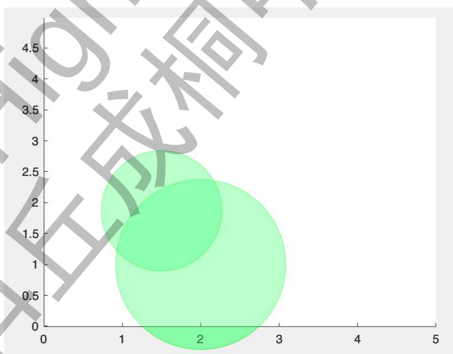

Figure 6: Results 3.1

As shown in the figure above, when no trade occurs, Civilization 1, the weakest, can be destroyed by any one of the two stronger civilizations. The two stronger civilizations will choose to cooperate together.

Now, we will introduce resource specialization to the community. Each civilization will have one resource that it specializes in, of which its price is extremely low. The other two civilizations use the resource extensively but has a significantly higher price, so they must rely on trade to acquire these resources. The list of resources and unit price is shown in Table 8; the results of the updated simulation is shown in Figure 8.
When trading between civilizations is allowed, the three civilizations stop to antagonize each other, despite not participating in information sharing, because through trading, they can earn the price difference between resources without war cost. If they control the resources by war, price levels will fall immensely and they will earn much less revenue. Thus, through trading and specialization, peaceful cooperation

Civilization	Resource 1	Resource 2	Resource 3
1	1	10000	10000
2	10000	1	10000
3	10000	10000	1

Table 6: List of Resources for Simulation 3 (Trade)

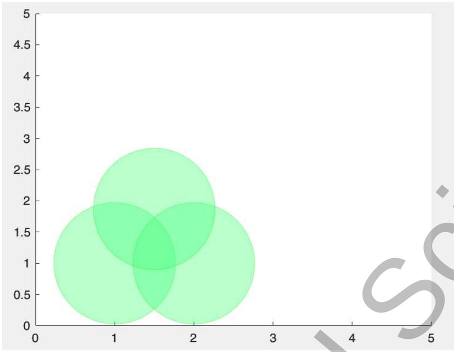

Figure 7: Results 3.2

between civilizations, even of different dimensions can be reinforced.

Nevertheless, the prospect of trading is limited to very special circumstances. In order to maintain peace, population levels have to be extremely low to prevent initiatives to take over civilizations for acquiring human resources. In addition, the lower price level has to be relatively close to 0 such that the loss of participating in war is statistically significant. Finally, the distance between these two civilizations must be within some range so that the cost of commerce does not increase exponentially.

7 Discussion

In this paper, we have discussed and experimented somewhat comprehensively with the topic of metaverse interactions. We began by exploring the possible motives why civilizations may choose to interact, organizing them according to their valance categories. Then, we listed a multitude of factors, such as population, dimension, and the Kardashev scale that may contribute to deciding the optimal interaction behavior. We used a variety of mathematical models, from solving exponential function to normal distributions to model equations of cost and benefit. Numerical analysis is the most important part that this paper advanced from previous attempts, since we are able to discuss situations with more accuracy.

Moreover, we also used the MATLAB programming language to complete simulation experiments, which helped us discover intriguing patterns realistically. According to our results analysis, our initial Dark Forest Hypothesis describes generally correct behavior, but is inaccurate in cases including but not limited to:

- When capabilities parameter is generally similar ($ecf < 0.8$), weaker civilizations may have the opportunity to merge and control the stronger civilizations;
- When the lower dimension civilization has a small population level (in the ten thousands range), higher dimension civilizations will not attack it; instead, cooperation will ensue;
- The extreme rarity and specialization of resources can encourage peace and collaboration;
- Most civilizations are not aggressive in nature: they act objectively and decide on the basis of what makes them benefit the most.

Undoubtedly, this paper has many limitations. Metaverses and virtual reality has been a relatively new field, and no hypothetical civilizations have been formally discussed or observed in virtual spaces , so there is a lack of data to analyze. When building the model, we overwhelmingly relied upon past events that happened on Earth. Thus, although the general patterns in this model are expected, there may be some minor inaccuracies. In addition, we assumed that Newtonian physics laws and the triangular inequality for planer distances still apply. Nevertheless, the universe is full of mysteries, and current physical laws can be violated via dark matter and black holes, which may significantly alter the results of this study.

Whilst we encourage future authors investigate unresolved issues in this paper, we would also like to underscore that the applications of this paper does not solely rely on a interstellar background. The methodology can also be applied to predict international relationships when the parameters are edited for this purpose. In the future where companies and communities become larger in size, game theory and relationships in general is likely to be a topic worth discussion.

8 Acknowledgements

This research paper represents the culmination of a long journey, one that would not have been possible without the support and guidance of several individuals and institutions. I would like to take this opportunity to express my deepest thanks to those who have played an essential role in the development of this research paper.

First and foremost, I would like to thank my mentor, Maximilien MacKie, a PhD candidate in mathematics at Hertford College, University of Oxford. Despite his expertise in pure and applied mathematics, Mackie was interested in my topic from the beginning and helped me voluntarily without charge. Mackie helped me with finding appropriate literature, offering suggestions on my previous approaches such as evolutionary game theory, but he encouraged me to complete my calculations independently. In particular, MacKie discussed this paper with me often and helped me eliminate unnecessary characteristics and simplify this model is crucial to this project, which need to be executed within reasonable time. His ability to distill intricate mathematical concepts into digestible and clear explanations allowed me to develop the foundation necessary for tackling the challenging modeling scenarios this paper explores. Maximilien's mentorship extended beyond the technical aspects of the research; he encouraged me to be patient with the process, embrace moments of doubt, and remain resilient in the face of setbacks. I am immensely grateful for his support, which has been instrumental in shaping not only the research but also my growth as a student of mathematical modeling.

I would also like to express my sincere thanks to YK Pao School for providing the academic environment that nurtured my curiosity and passion for research. The school's dedication to academic rigor and holistic development has empowered me to pursue ambitious projects such as this one. Special thanks are due to my teachers, who have fostered my interest in mathematics and complex systems, and to the school's external competition coordinator Yifan Shen, whose guidance helped me understand the basics of academic research. YK Pao's emphasis on critical thinking and independent inquiry played a fundamental role. The opportunity to explore topics outside of the conventional curriculum allowed me to delve into areas of interest, culminating in the formation of my research question on inter-civilizational communication in virtual and real-world contexts.

Reflecting on the research process itself, I am grateful for the intellectual freedom I was afforded to explore this difficult topic, since many view this as impossible to investigate for high school students with the lack of empirical data and reliability. The question I sought to answer—how civilizations, including virtual and extraterrestrial ones, could communicate across dimensions and technological capabilities—emerged from a long-standing fascination with both virtual worlds and space exploration. Initially inspired by the growing significance of platforms like META and longstanding SETI efforts, I sought to merge these two seemingly disparate fields through the lens of game theory. However, the path from formulating the question to developing a rigorous mathematical model was far from straightforward.

The process began with a literature review, which helped me familiarize myself with the current state of research in both virtual interactions and interstellar communication. Drawing connections between these two realms was challenging, as existing research often treated them separately. I gradually developed the framework that combined game theory, equation modeling, and probabilistic Markov chains to simulate communication scenarios between civilizations. The early stages of building the mathematical model were daunting, as I struggled to balance realism with abstraction in the equations.

There were moments of difficulty, especially in reconciling the stochastic nature of communication with the deterministic structure of game theory models. At one point, I faced a major roadblock when the equations I had developed yielded inconsistent results, leading me to question whether the model was fundamentally flawed. It was during this time that the encouragement of my mentor, my school, and my family became particularly vital. Maximilien helped me reevaluate the assumptions behind my equations, ultimately guiding me toward a more robust model. My teachers at YK Pao also provided a supportive environment, offering both technical advice and moral support, which helped me regain confidence in my

work.

I would be remiss if I did not extend my deepest gratitude to my parents, who have been my greatest supporters throughout this process. Their unwavering belief in my abilities, coupled with their encouragement to pursue my intellectual passions, has been the bedrock of my academic journey. They provided me with the time, space, and resources to fully immerse myself in this research. Their emotional and practical support has been invaluable, and I am incredibly fortunate to have them as my foundation.

References

[1] Charles Adler. "Wizards, Aliens, and Starships: Physics and Math in Fantasy and Science Fiction". In: *Princeton University Press* (2019).

[2] Slikker M. et al. "Information sharing games". In: *International Game Theory Review* (2003). DOI: 10.2139/ssrn.249036.

[3] *Aliens, the Fermi Paradox, and Dark Forest Theory*. blogs.cornell.edu/info2040/2021/09/28/aliens-the-fermi-paradox-and-dark-forest-theory-a-game-theoretic-view/.

[4] *Anti-Japanese War of Resistance - World War II Chinese Theater*. www.csun.edu/~yz73352/ChinaReunite/1937-1945/anti-Japanese.htm.

[5] *Markov Chain - Earth and Planetary Sciences*. https://www.sciencedirect.com/topics/earth-and-planetary-sciences/markov-chain. 2021.

[6] *Military production during World War II*. https://www.ukpublicspending.co.uk/past_spending.

[7] *NASA: Cost of Apollo missions 1968-1972*. www.statista.com/statistics/1028322/total-cost-apollo-missions/.

[8] *Three-Body Review*. www.nytimes.com/2023/02/03/arts/television/three-body-review.html/. 2023.

[9] Sharp Tim. *How Far is the Moon?* www.space.com/18145-how-far-is-the-moon.html.

[10] *What is the Metaverse?* https://about.meta.com/what-is-the-metaverse/. 2024.

[11] Steele Zia. *Fictional Civilizations and the Kardashev Scale — Types II and III*. medium.com/whiteboard-to-infinity/fictional-civilizations-and-the-kardashev-scale-types-ii-and-iii-d9184edb9c5.