参赛学生姓名：聂关晏

中学：深圳国际交流学院

省份：广东省

国家/地区：中国，南方赛区

指导老师姓名：吴邵汐

指导老师单位：深圳国际交流学院

论文题目：Bi-level game analysis on maritime supply chains under vessel speed reduction policies

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# Bi-level game analysis on maritime supply chains under vessel speed reduction policies

Nie Guanyan

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**Abstract**

This study examines the implementation of emission regulations on maritime supply chains. In particular, we first show how various operational variables of shipping lines can determine the successful maintenance of a Vessel Speed Reduction Subsidy (VSRS) through an evolutionary game of a port and a shipping company. Then, we model the effects that implementing a VSRS has on shipping companies and ports under different competition structures through a bi-level dual-channel game model while the optimal decision variables for each player are computed. Through this game, it is found that implementing VSRS can decrease the port's emissions while increasing emissions at the competing port with varying magnitudes of effect under different competition structures between shipping companies. Moreover, implementing such a policy also improves the shipping line's financial performance whilst worsening that of the competing shipping line. Later, a complimentary numerical analysis is performed accordingly to illustrate and explain these results in detail. Subsequently, to include the governmental policies into consideration, we extend the game to examine the effects of a subsidy-sharing policy on ports and demonstrate how it improves both shipping lines' financial performance.

**Keywords**: Maritime Supply Chains, Vessel Speed Reduction Policy, Evolutionary Game Theory, Supply Chain Management, Emission Regulations

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# 1 Introduction

With growing concerns over climate change and the push for green initiatives, the maritime transportation industry has come under increased scrutiny by researchers and policymakers alike due to the significant emissions it produces. Over merely the last decade, the industry's greenhouse gas (GHG) emissions, have risen over 20% to now producing a total of well over 800 million tons of $CO_2$ emissions in 2023 [1]. Continuing the current pace of growth, IMO predicts GHG emissions to increase 90-130% by 2050 compared to 2008 levels. In 2016, by the signing of the Paris Agreement, nations pledged to decrease $CO_2$ by 45% in 2030 and become net zero by 2050. Hence, at the current rate, without more attention being put towards this issue, these promises can only surrender to futility.

By accounting for over 90% of global trade, the maritime transport industry provides the foundation that economies nowadays are built upon. However, as COVID-19 has exposed, the fragility of maritime supply chains and the devastation it wreaks on economies upon breakage suggests the urgent need for better management practices to reinforce this foundation. This highlights the necessity for a balanced approach to re-configuring the industry's operations, ensuring that the environmental goals are met whilst also not undermining supply chain resilience and economic consequences. Positioned at the junction of multiple concerns in the current decade such as global warming, increased occurrences of supply chain disruptions, the US-China trade war, and escalating geopolitical tensions, this industry asserts itself as one of the most crucial areas of research in current times.

## 1.1 Related works

Due to these persisting conflicts, vessels have been forced to reroute and reschedule their voyages, causing significant uncertainty in the shipment schedule for ports. With an overwhelming surplus of demand at certain ports, severe congestion is seen amongst them, leading to all kinds of supply chain issues globally. To combat this, it is crucial to reduce this uncertainty. The most obvious solutions, as proposed by previous literature are improving this system physically: increasing port capacity, capital investments, [2] and other policies such as capacity-sharing or using cross-port investments [3]. However, these strategies are only viable in the long term due to their need for either physical alteration to the maritime supply chains or substantial investments.

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Adding to the strain on these tenuous maritime supply chains, environmental regulations have added another layer of financial burden and operational restraint to this industry. Effective implementation of emission regulations plays a crucial role in successfully abating these environmental aftermaths [4] while also not encumbering their operational performance.

Greenhouse gas (GHG) emissions are well-known to cause all kinds of health and climate problems, however, its economic costs have yet to be fully understood. Due to the economic impacts of these emissions being dynamic in nature [5], often remaining latent in the short term, we often underestimate their magnitude and severity. To address this global issue, a multitude of strategies and regulations have been proposed in the past, ranging from mandatory regulations to industry practices [6] [7]. The International Maritime Organization (IMO) has also set stringent regulations requiring the adoption of low sulfur fuels (LSF) and very-low sulfur fuels (VLSF) and exhaust gas cleaning systems (EGCS) [8]. Many studies also discuss the adoption of alternative fuels that produce lower GHG upon combustion [9, 10]. Michaela et al. [11] through a life-cycle-analysis (LCA) compared the emission of fossil fuels and biofuels. However, complying with such stringent emission regulations has been shown to bring additional financial risks, leading to challenges in meeting consumer demand and lowered profitability [12]. This has therefore prompted scholars to bridge the field of financial risk optimization and maritime emission regulations. Using an ambiguous robustness optimization model and a mixed-integer model, Sun et al [13] propose a decision system to minimize the financial riskiness index of shipping companies' technology investments. Gu et al. [14] investigate the effect of bunker risk management methods such as fuel hedging and risk aversion strategies affect the CO2 emission of shipping companies. However, these strategies and policies studied often provide shipping companies with no flexibility by forcibly adding extra costs to their operation due to having to adhere to these regulations, neglecting the immense stress they are already being put under by the wider industry headwinds and uncertainties.

Therefore, in this study, we decide to examine specifically the Vessel Speed Reduction Program (VSRP) where vessels can optionally choose to slow down to benefit from a subsidy grant by the port for reducing emissions in the area. This approach unlike the aforementioned strategies provides not only the shipping companies the option to slow down or not at their own decision but also the ports and governments to fine-tune the implementation parameters of this program such as the subsidy amount, the speed limit, and more.

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Following past literature [15, 16], fuel consumption can be represented as a cubic function of the vessel speed. Since fuel costs contribute to 40-67% of total shipping costs [17], reducing speed can significantly reduce operating costs for shipping companies. To provide a perspective of how much this is, we can take the example of a Panamax vessel, one of the most popular vessels in the current maritime fleet. Such vessels carry up to 5000 TEU and operate on average around 23 knots (1 knot $\approx$ 0.51 m/s) [1]. According to data from Goicoechea et al. [18] and the IMO [19], one vessel of this model consumes approximately 44643.69 tons of fuel per year. With a heavy fuel oil price of USD 428/ton (as of December 2023), this amounts to annual fuel costs of over 19 million USD. By merely reducing consumption by 5%, we can reduce the annual costs by over 950 000 USD.

Likewise, emissions, which are directly linked to fuel consumption should also follow a cubic relationship with vessel speed. Indeed, previous literature has demonstrated the effectiveness of speed reduction of vessels in reducing emissions [20] [21]. Doing so also leads to improvements in aspects such as energy efficiency [22], port accident mitigation [23], and underwater noise levels that harm marine animals [24]. Despite the growing interest in the benefits of vessel speed reduction, it is still not enough to encourage vessels to slow down voluntarily, hence prompting further research into the optimal implementation of subsidized vessel speed reduction policies, or as we hereby call it, the Vessel Speed Reduction Subsidy (VSRS).

## 1.2 Contributions of this paper

Firstly, we analyze how maritime ports' operational decisions can impact the implementation of VSRS policy over time. We build an evolutionary game theory model to analyze how these decision variables affect the implementation of VSRS and port emissions. With the effectiveness of VSRP-type strategies in reducing emissions evidently proven in past literature and practice, it is crucial for us to examine how to successfully implement such strategies. Through an evolutionary approach, we can analyze how the implementation of VSRS changes over time, determining the conditions in which such a strategy can be sustained and managed successfully. We consider a two-player game, involving a port and a shipping company. The port can choose to implement VSRS or not implement it whilst the shipping company can choose to or not to adhere to the speed limit under both scenarios. Through employing a replicator dynamic mechanism, we then

[1] https://www.inboundlogistics.com/articles/post-panamax-ships/



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find the evolutionarily stable strategies and analyze how different factors may affect them. We find that many factors influence the shipping company's choice of action whilst only one (the additional revenue produced from new vessels docking at the port) noticeably affects the choice of port. Also, unlike other emission regulation policies, subsidy amounts being higher than a certain threshold may instead lead to failure in maintaining the VSRS policy over time. Subsequently, a numeric simulation is performed to illustrate these effects and trends, aiding in further analyzing the results.

Contrary to other emission reduction policies for the maritime industry where higher subsidies directly translate to lower emission [25] [26], subsidy amount being higher than a certain threshold may instead lead to a failure in maintaining the VSRS policy over time, hence instead increasing the emission.

Secondly, we also analyze how the implementation of VSRS in a bi-level dual-channel shipping line affects the operations of ports and shipping companies, considering different market competition structures. We develop a similar game as the previous evolutionary game part but with 2 ports, each with a corresponding shipping company. Since our goal in this section is to study the impacts of VSRP on the shipping lines, we will constrain this part to a static game instead of following the evolutionary game approach. Past literature either considers this problem from only the perspective of one player or studies it empirically on a single shipping line. Moreover, strong assumptions of the competition structure between entities in shipping lines could lead to invalid results when real markets deviate from such an assumption. Hence, in our two-stage hierarchical game model with two shipping lines, one is modeled as a Nash-Nash game whilst the other as a Nash-Stackelberg game. Port 1 implements a VSRS while Port 2 does not. Consequently, vessels of Shipping Company 1 (corresponding to Port 1) adhere to a vessel speed limit when within the speed reduction zone (SRZ) whilst Shipping Company 2 vessels operate at their usual speed. Through mathematical derivations of our game, we find the optimal operational variables for shipping lines (ports and shipping companies), including the freight rates, port service charges, and quantity shipped. We also analyze the maximum profits and minimum emissions achievable by the ports and shipping companies. We then analyze how implementation details of VSRS may affect the aforementioned variables. Complimentary, a numerical analysis is then performed, visualizing clearly how VSRS affects shipping lines.

Thirdly, to further the study on VSRS, we extend our game model to consider also the govern-



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ment in a tripartite game and study how the government intervention impacts the game's results. Specifically, we consider a subsidy-sharing policy where the government covers a share of the subsidy given out by the port to support the VSRS. Examining the additional effects of this government policy, we find that it noticeably improved shipping lines' financial performance.

The remainder of the paper will be organized as follows: Section 2 will construct an evolutionary game model and analyze how the operational decisions of ports and shipping companies evolve over time. Section 3 builds a similar but hierarchical dual-channel game, modeling shipping lines' operations under VSRS, considering both a Nash-Nash game and a Nash-Stackelberg game. Optimal operational variables are computed and a numerical analysis is performed subsequently. Section 4 extends the model from Section 3 to now consider the role of government intervention in improving the implementation of VSRS. Section 5 provides a conclusive review of this study and proposes potential areas for future studies.

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Table 1: Variable definitions

| Symbol | Defintions |
|---|---|
| $v_i$ | Velocity of ships from Company i |
| $v_s$ | Velocity limit in speed reduction zones |
| $p_i$ | Freight rate charged by Company i |
| $w_{pi}$ | Docking fee charged by Port i |
| $Q_i$ | Quantity of products shipped by Company i per year |
| $Q$ | Total products shipped per year |
| $\eta$ | Subsidy amount paid for ships adhering to speed reduction policy |
| $t_i$ | Total time spent sailing for ships from Company i per trip |
| $t_s$ | Time spent sailing for ships in speed reduction zones |
| $d_i$ | Total distance travelled by ships from Company i |
| $d_s$ | Distance travelled in speed reduction zones |
| $p_f$ | Price of fuel |
| $\alpha$ | Storage costs per unit quantity per unit time during shipping |
| $\beta$ | Congestion-related costs per unit quantity |
| $w_i$ | Storage capacity of ships from Company i |
| $r_i$ | Fixed costs of ships from Company i |
| $T_i$ | Average time spent sailing for ships from Company i per year |
| $\pi_i^S$ | Profit of shipping company i |
| $\pi_i^P$ | Profit of port i |
| $k$ | Fuel efficiency |
| $x$ | Probability of port choosing to implement VSRS |
| $y$ | Probability of shipping company choosing to slow down |
| $F_n$ | Annual fuel consumption in SRZ with normal speed |
| $F_s$ | Annual fuel consumption in SRZ with reduced speed |



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Table 2: Variable definitions (continued)

| **Symbol** | **Defintions** |
|---|---|
| $N$ | Number of vessels served by the port |
| $n_{trips}$ | Number of trips per year per ship |
| $C_1$ | Costs of increased emissions |
| $C_2$ | Opportunity cost of slowing down due to time costs |
| $C_3$ | Opportunity cost of not slowing down due to subsidy forgone |
| $R_0$ | Base reward for port without implementing VSRS |
| $R_1$ | Additional rewards for port from implementing VSRS |
| $R_2$ | Additional rewards from re-routed vessels |
| $R_p(x)$ | Replicator dynamics function for port |
| $R_s(y)$ | Replicator dynamics function for shipping company |
| $E_{0i}$ | Emission at port i with VSRS not implemented |
| $E_i$ | Emission at port i with VSRS implemented |
| $z$ | Proportion of subsidy shared by the government |
| $\lambda$ | Pollution coeffcient |
| $F_{di}$ | Total fuel consumption of shipping company during docking |
| $f_{di}$ | Fuel consumption shipping company during docking per unit time |
| $F_{si}$ | Total fuel consumption of shipping company in SRZ |
| $C_G$ | Government's costs regarding the two shipping lines |
| $\Delta C_s$ | $p_f k(v_s^2 - v^2) + (\alpha w + \frac{r}{T})(\frac{1}{v_s} - \frac{1}{v})$ |
| $\phi_1$ | $\frac{k d_s v_s^2}{w}$ |
| $\phi_2$ | $\frac{p_f k d_s v_s^2}{w} + \frac{r d_s}{w T v_s}$ |
| $\phi_3$ | $\frac{d_2 - d_1 + d_s}{2v} \times (\alpha \frac{r}{wT}) - \frac{(\phi_2 + \alpha t_s)}{3}$ |
| $\theta_6$ | $\frac{k d_s v_s^2 + t_d f_d}{w}$ |
| $\theta_7$ | $\frac{k d_s v_{2_{+t_d f_d}}}{w}$ |



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Table 3: Numerical values of variables

| **Variable** | **Description** | **Value** |
|---|---|---|
| $N$ | Number of vessels served by the port | 300 |
| $n_{trips}$ | Number of trips per year per ship | 7 |
| $C_2$ | Opportunity cost of slowing down | $1.4e^7$ |
| $C_3$ | Opportunity cost of not slowing down | $2.5e^6$ |
| $R_2$ | Additional rewards from re-routed vessels | $6.0e^6$ |
| $p_f$ | Heavy Fuel Oil (HFO) price | \$428.0/ton |
| $k$ | Engine efficiency | $5.42 \times 10^{-4}$ |
| $\alpha$ | in-transit inventory cost | \$0.87/h/TEU |
| $w$ | Veseel capacity | 6000 TEUs |
| $T$ | Average working time at sea annually | 6480 h |
| $\beta$ | Congestion cost coefficient | $10^{-4}$/TEU |
| $Q$ | Annual demand | $10^6$ TEU |
| $r$ | Fixed cost coefficient | $2.7 \times 10^6$ |
| $\eta$ | Subsidy amount per unit of fuel consumed | \$300/ton |
| $c$ | Marginal costs of port service | \$180/TEU |
| $d_1$ | Distance travelled by vessels of company 1 | 2000 nm |
| $d_2$ | Distance travelled by vessels of company 2 | 2000 nm |
| $d_s$ | Radius of SRZ | 50 nm |
| $v_s$ | Speed limit in SRZ | 10 nm/h |

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# 2 Evolutionary game of ports and shipping companies

## 2.1 Model setup

In this section, we build an evolutionary game theory model to analyze the impact of VSRS on ports' and shipping companies' operational decisions. In this game, there exists a port and a corresponding shipping company that is assumed to always offload at this port. The port can choose either to implement the VSRS or not implement it. The shipping company on the other hand can choose to: 1. Reduce their vessel speeds (to the speed limit $v_s$ regardless of whether a VSRS is implemented at the port or not, 2. Not reduce their vessel speeds. The implementation of the VSRS consists of 3 parameters: $v_s$, the speed limit in the speed reduction zone (SRZ); $d_s$, the radius of the SRZ; $\eta$, the subsidy amount.

Considering this problem from the port's perspective, we list out its costs and rewards in order to produce a payoff matrix. For the base case, we assume that the port does not implement a VSRS and the shipping company does not reduce their vessel speeds. By not implementing the VSRS, the port achieves a base reward of $R_0$; by implementing the VSRS, the port obtains an additional reward of $R_1$. Through the addition of the VSRS, the port may also attract new vessels that have rerouted to take advantage of this subsidy, hence granting the port another additional reward of $R_2$.

From the shipping company's perspective, if the vessels choose not to slow down in the SRZ set by the port, they will incur a cost of $C_1$ on the port due to the increased emission sailing produced at higher speeds. The fuel consumption follows a cubic relationship with the vessel speed $v$, hence we here represent it as $kv^3$, where $k$ is the fuel efficiency. In this case, the annual fuel consumption by all vessels of the shipping company $F_n = N_n \times kv^3 \times \frac{d_s}{v} \times n_{trips} = Nkv^2 d_s n_{trips}$, where $N$ denotes the number of vessels served by the port annually and $n_{trips}$ denotes the average number of trips taken by each vessel annually. The shipping company will also incur an opportunity cost of $C_3$ on themselves, representing the total subsidy amount that they would have received for slowing down in a year.

If the vessels choose to slow down, the annual fuel consumption will then be expressed as $F_s = Nkv_s^2 d_s n_{trips}$. However, having a slow delivery speed can cause dissatisfaction in the customer and thus lower their willingness to pay, having detrimental effects on the revenue of the shipping company (time costs) [27]. Therefore, in this scenario, the shipping company will incur an opportunity cost

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of $C_2$ upon itself, representing the annual profits lost due to shipping at a slower rate.

Hence, now we can compute the payoff matrix of this game:

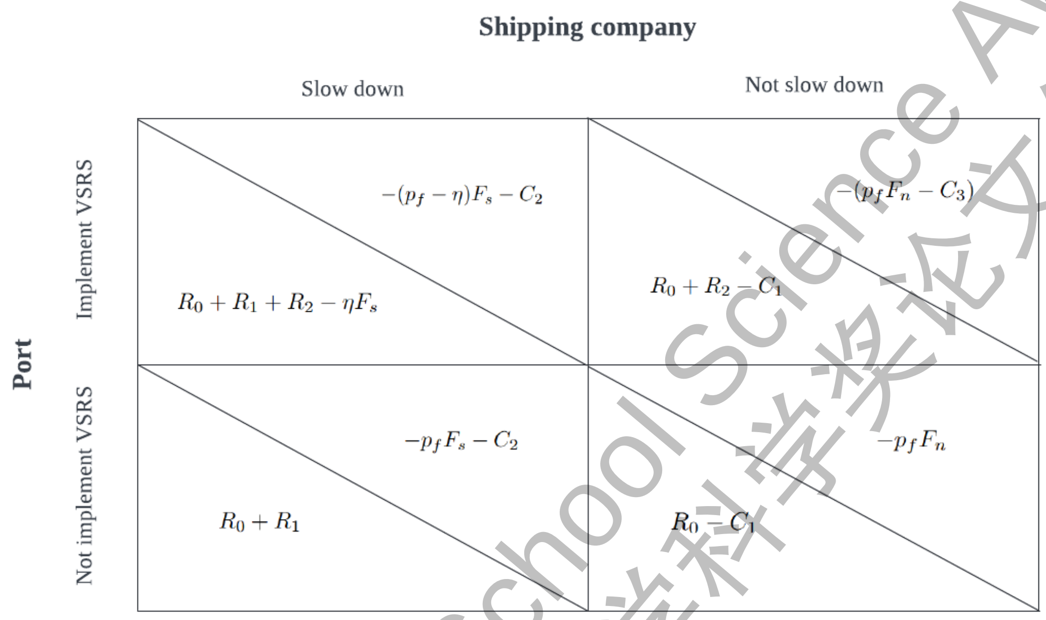


Figure 1: Payoff matrix of the evolutionary game

Assuming that the port chooses to implement the VSRS with a probability of $x(0 \leq x \leq 1)$ and not to implement the VSRS with probability $(1-x)$. Similarly, the shipping company will choose to slow down until under the speed limit with a probability of $y$ and not slow down with a probability of $(1-y)$. When we consider from the port's perspective, when the port adopts the "Implement VSRS" strategy, the expected payoff $P_{11}$ is as follows:

$$P_{11} = y(R_0 + R_1 + R_2 - \eta F_s) + (1-y)(R_0 + R_2 - C_1) \tag{1}$$

When the port adopts the "Not implement VSRS" strategy, the expected payoff $P_{12}$ becomes:

$$P_{12} = y(R_0 + R_1) + (1-y)(R_0 - C_1) \tag{2}$$

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Hence, we can derive the expected payoff $P_{11}$ of the port as shown below:

$$\bar{P}_1 = x(R_2 - y\eta F_s) + y(R_0 + R_1) + (1-y)(R_0 - C_1) \tag{3}$$

By following the same procedure as above from the shipping company's perspective, we can get:

$$P_{21} = x\eta F_s - p_f F_S - C_2 \tag{4}$$

$$P_{22} = -xC_3 - p_f F_n \tag{5}$$

$$\bar{P}_2 = y(x\eta F_s - p_f F_s) + (1-y)(-xC_2 - p_f F_n) \tag{6}$$

Assuming uniform population distribution, we use the replicator equation to find the replicator dynamic equations of the port and shipping company to be as follows:

$$R_p(x) = x(1-x)(R_2 - y\eta F_s) \tag{7}$$

$$R_s(y) = y(1-y)[x(\eta F_s + C_3) + p_f(F_n - F_s) - C_2] \tag{8}$$

## 2.2 Model results

According to the replicator dynamic, for the port to achieve an evolutionarily stable strategy, $R_p(x)$ must be equal to 0 and $R'_p(x)$, the derivative of $R_p(x)$ must be negative. Taking the derivative of $R_p(x)$, we get $R'_p(x) = (1-2x)(R_2 - y\eta F_s)$. When $R_2 > \eta F_s$, $R'_p(0) > 0$ and $R'_p(1) < 0$, hence $x = 1$ is the evolutionarily-stable strategy in this scenario. Hence, the port must ensure that the total subsidy amount they provide to shipping companies ($\eta F_s$) is less than the extra revenue received from rerouted ships ($R_2$) in order for them to be able to continue to implement the VSRP stably. Therefore, we can infer that giving an overly large sum of subsidy may not help with the implementation of the VSRP backfire and increase a port's emissions.

When $R_2 \leq \eta F_s$, we need to consider three cases: Firstly, when $y = y^* = \frac{R_2}{\eta F_s}$, no matter the value of $x$, $R_p(x)$ and $R'_p(x)$ are both always equals to 0. Therefore, in this scenario, the strategy will not evolve but instead stay constant as illustrated in Figure 2. We term the $y$ value in this case as the evolutionary stable value $y^*$.

Secondly, when $y < \frac{R_2}{\eta F_s}$, $F'_p(0) > 0$ and $F'_p(1) < 0$, hence $x = 1$ is the evolutionarily-stable

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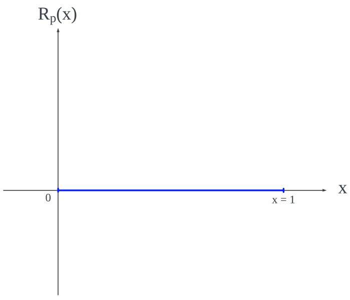


Figure 2: Evolution of $x$ when $y = y^*$

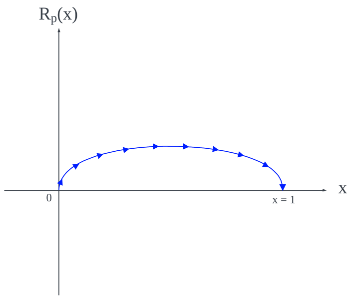


Figure 3: Evolution of $x$ when $y < y^*$

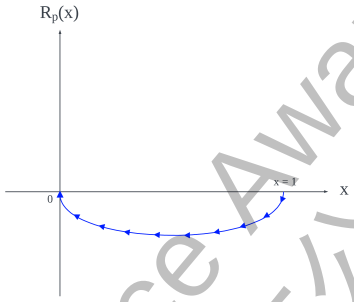


Figure 4: Evolution of $x$ when $y > y^*$

strategy in this scenario, meaning that no matter the initial value $x$, it will evolve and move towards 1 as shown in Figure 3.

Thirdly, when $y > \frac{R_2}{\eta F_s}$, $F_p'(0) < 0$ and $F_p'(1) > 0$, thus $x = 0$ becomes the evolutionarily-stable strategy in this case as seen in Figure 4.

Therefore, we can see that if the total subsidy granted goes beyond the reward received coming from newly rerouted vessels, the port must ensure that the subsidy amount is not too large so that the value $\frac{R_2}{\eta F_s}$ is still larger than probability $y$ that the shipping company chooses to slow down. If not so, the port would then lose control over the evolutionary dynamics of its strategy since whether implementing the VSRP is an evolutionarily stable strategy now depends on the probability $y$ of the shipping company. Hence, to mitigate this unnecessary risk, ports should always set their total subsidy amount lower than the $R_2$ value. In practice, further models can be developed to estimate this value of $R_2$ and hence provide the port a concept of approximately how to set their total subsidy amount.

Through our analysis above, we can arrive at the following proposition about the port's choice of strategy in this evolutionary game:

**Proposition 1** *: The evolutionary dynamics of the port's strategy are not affected by the port's base reward $R_0$, or in other words, its initial operating profits, the environmental costs $R_1$ saved by implementing VSRS, and also the pollution costs $C_1$ incurred by the shipping company for not slowing down. It is however largely dependent on the additional revenue $R_2$ generated by ships that have rerouted to the port and also the subsidy amount $\eta$.*

The same analysis procedure can be done for the shipping company. Due to it being overly

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repetitive, we will skip it for this section. Nevertheless, we can easily elaborate upon it graphically alongside the previous analysis on the ports in the subsequent simulation with numerical values performed in the following sub-section.

## 2.3 Simulation

Using MATLAB to model the replicator dynamics equations, Figure 5 illustrates how the two probabilities $x$ and $y$ evolve over time with different initial values. For estimating the specific values of $F_n$ and $F_s$, we continue to refer to the aforementioned Panamax ship model where firstly $v$ is set to 23 knots (nautic miles per hour). For container ships of this size, an average of 203.4 tons of bunker fuel is consumed per day when traveling at a speed of 25 knots [28]. This information calculates the hourly engine efficiency where $k = 203.4 \times 25^{-3}/24 = 5.42 \times 10^{-4}$. Following current implementations of vessel speed reduction programs, the reduced speed limit $v_s$ is set to 10 knots [2]. Hua et al. [29] note that Post Panamax vessels, which are larger than Panamax vessels, travel on average 5 round trips per year each. Hence, we will set $n_{trips}$ to be 7 due to Panamax vessels' smaller sizes. Using the data above, we can compute the values for $F_n$ and $F_s$ to be 30105.39 and 5691, respectively. Without loss of generality, we set $d_s$ to be 50 nautic miles, both $N_n$ and $N_s$ to be 300, $\eta$ to be \$150USD, $R_2$ to be $6.0e^6$, $C_2$ to be $1.4e^7$, and $C_3$ to be $2.5e^6$.

From the results in Figure 5, we see that no matter the initial probabilities of implementing VSRS ($x$) and the speed reduction of shipping companies ($y$), the port will always evolve to favor implementing VSRS while the shipping companies will always move towards not slowing down their vessels.

### 2.3.1 Sensitivity analysis

(1) The effect of fuel price on the evolutionarily stable strategies

One of the largest uncertainties faced by the maritime industry is the highly volatile fuel prices. Especially due to the sustained geopolitical conflicts occurring in or near large oil-producing countries, this aspect poses a significant risk to maritime supply chains. Setting varying values for fuel

[2]Multiple ports in the US (e.g. Los Angeles, San Diego, New York) implement vessel speed reduction policies with speed limits between 10 to 15 knots: https://www.epa.gov/ports-initiative/marine-vessel-speed-reduction-reduces-air-emissions-and-fuel-usage. Policy implemented by the Greater Farallones and Cordell Bank national marine sanctuaries in the US for protecting whales also uses a similar speed limit of 10 knots: https://farallones.noaa.gov/eco/whales/vessel-speed-reduction.html



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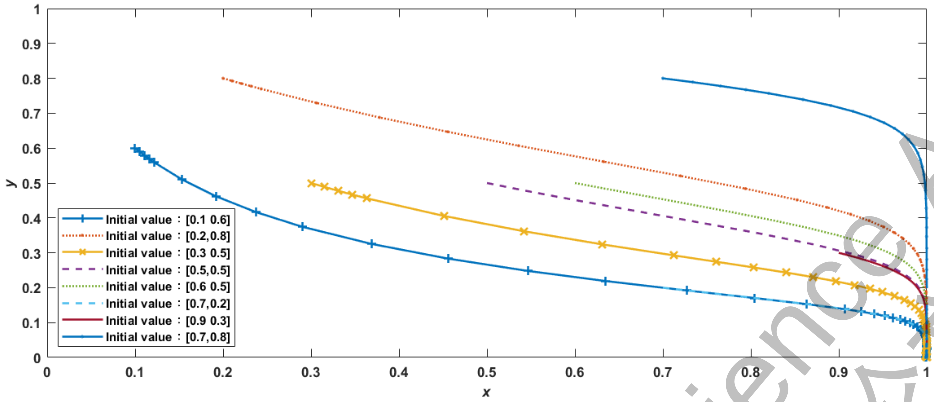


Figure 5: Simulation results of the evolution of $x$ and $y$

price $p_f$, ceteris paribus, Figures 6 and 7 demonstrate the simulation results of how $x$ and $y$ evolve over time, respectively. The "time" on the x-axis represents the iterations of evolution and hence does not have a specific unit. From the results, we see that the price of fuel actually does not influence the stability of these evolutionarily stable strategies, having no effect on $x$ whatsoever and only slightly affecting the speed at which $y$ converges to 0.

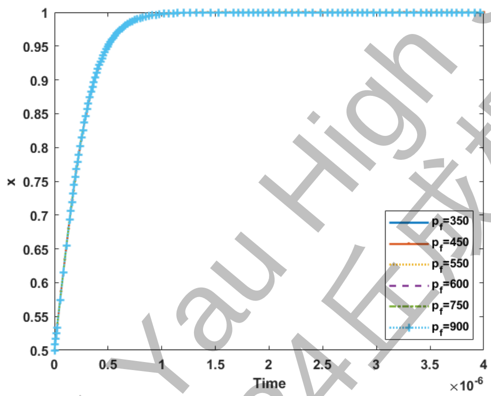


Figure 6: Effect of different fuel prices on the evolution of $x$

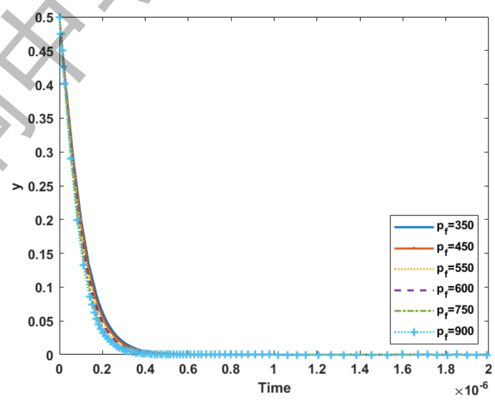


Figure 7: Effect of different fuel prices on the evolution of $y$

(2) The effect of subsidy amount on the evolutionarily stable strategies

With other variables staying the same as the base case when $\eta$ = 50, 100, 150, 200, 250, 300 and when the initial probabilities are set to $(x = 0.5)$ and $(y = 0.5)$, Figures 8 and 9 depict the

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simulation results. For the case of $x$, a higher subsidy amount $\eta$ slows down the speed at which $y$ moves towards 1, hence reflecting how ports may be hesitant to implement the policy if they have to give out a large amount of subsidy. For the case of $y$, we can see that $\eta$ has a moderating effect on the evolutionarily stable strategies where for $\eta$ greater than a value somewhere between 100 and 150, the shipping company evolves to choose the strategy "slow down" whilst, below this value, the shipping company evolves to choose the strategy "not slow down". Initially, note that there is a small period in which a few instances of $y$ decreased marginally before increasing to 1. This can be attributed to how it takes time for $x$ to increase. During that period, the values of $x$ may be too low for the shipping company to choose to slow down, hence they move initially towards $y = 0$.

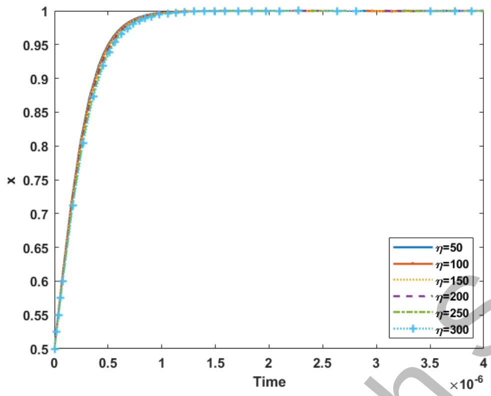


Figure 8: Effect of different subsidy amounts on the evolution of $x$

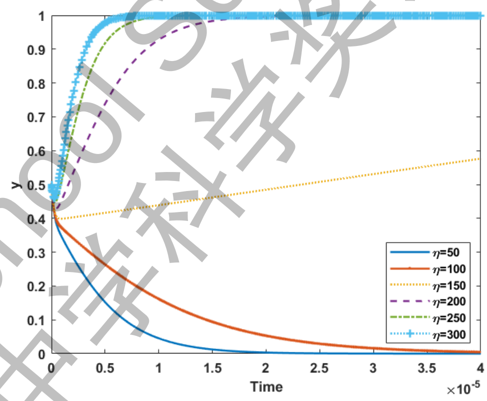


Figure 9: Effect of different subsidy amounts on the evolution of $y$

(3) The effect of $R_2$ on the evolutionarily stable strategies

With other variables staying the same, when $R_2 = 0$, $1.0e^5$, $5.0e^5$, $1.0e^6$, $1.5e^6$, $2.0e^6$ and the initial probabilities set to $(x = 0.5)$ and $(y = 0.5)$, Figures 10, 11 depicts the simulation results. As proposed in Proposition 1, the reward $R_2$, unlike other variables, can induce an influence on the evolutionarily stable strategy of the port. From Figures 10 and 11, we see that certain values $R_2$ cause the values of $x$ and $y$ to move in periodic oscillations between 0 and 1. Analyzing Equation 7, we can see that when initially $R_2$ is set above $y\eta F_s$, this will make the replicator function of $x$ positive, hence increasing the value of x in the next iteration. As $x$ grows larger, the term $x(\eta F_s + C_3)$ in Equation 8 increases, implying that the benefits of reducing vessel speed now increase. This then leads to an increase in $y$, hence the term $y\eta F_s$ in Equation 7 will gradually increase to be larger



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than $R_2$. This will then initiate the opposite trend where $x$ will then decrease, and $y$ will then follow behind, repeating this cycle perpetually. The time lag between the increase in $x$ and $y$ is justified by the fact that it takes a few interactions for $x$ to increase to a high enough value to make $R_s(y)$ positive and vice versa when $x$ causes a decrease in $y$.

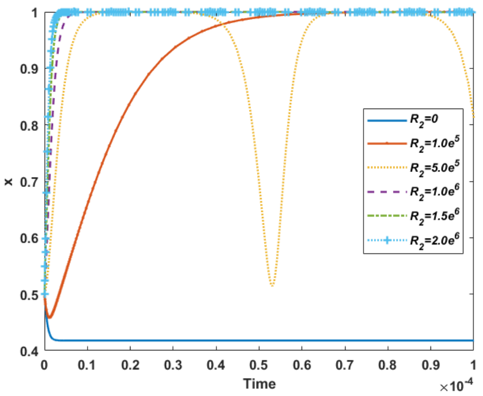


Figure 10: Effect of different $R_2$ values on the evolution of $x$

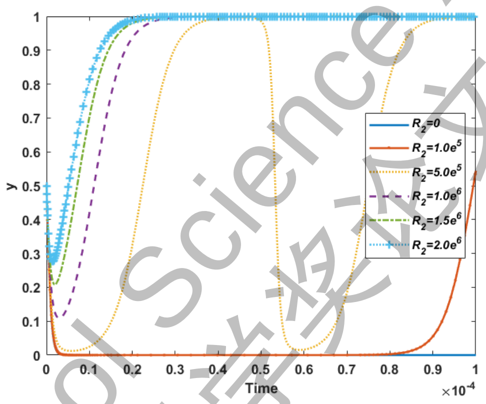


Figure 11: Effect of different $R_2$ values on the evolution of $y$

The simulation results for $C_2$ and $C_3$'s results are extremely similar to that of the fuel price $p_f$, except that $C_3$ does not affect $x$. The figures 24, 25, 26, 27 can be found in the Appendix section.

# 3 Impact of VSRS on shipping lines under different competition structures

Competition in maritime supply chains has evolved way beyond the level of individual components but rather entire sections of these logistic chains. Hence, it is crucial to consider the perspectives of agents in different stages of the supply chain. In our model, vessels from shipping company 1 only dock at port 1 whilst that of shipping company 2 only dock at port 2. Due to heavy vertical integration in maritime supply chains as noted by [30], we assume that shipping companies do not switch servicing ports in our model. We consider the situation where only Port 1 implements the VSRS whilst Port 2 does not. We model this problem as a hierarchical game involving two stages: First, the ports set their docking fees $w_{pi}$. Second, given the docking fees, the shipping companies would then determine their shipping prices $p_i$ for which they charge the shippers and

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the speed $v_i$ that their vessels operate at. In order to evaluate the problem more thoroughly and realistically, this paper considers two types of relationships between the two shipping companies: a Nash game where there is no difference between the companies' positions and a Stackelberg game where shipping company 1 is the leader and company 2 is the follower.

Let the total distance traveled by vessels of shipping company 1 per round trip be $d_1$ and the distance traveled in speed reduction zones (SRZs) specifically be $d_s$. Given that vessels of company 1 travel on average at a speed of $v_1$ outside of SRZs and $v_s$ inside them, the total time spent traveling $t_1 = \frac{d_1 - d_s}{v_1} + t_s$ where $t_s = \frac{d_s}{v_s}$. On the other hand, for shipping company 2, the total time spent traveling per round trip is simply $t_2 = \frac{d_2}{v_2}$.

Using the expressions of $t_1$ and $t_2$, we can represent the costs for shippers (usually the manufacturer or retailer) as follows:

$$C_1 = p_1 + \alpha(\frac{d_1 - d_s}{v_1} + t_s) + \beta Q_1 \quad (9)$$

$$C_2 = p_2 + \alpha(\frac{d_2}{v_2}) + \beta Q_2 \quad (10)$$

$p_1$ and $p_2$ represent the freight rates that shipping companies 1 and 2 charge their customers respectively. $\alpha$ denotes the storage costs coefficient, reflecting how much storing the cargo on board costs per unit of time. $\beta$ represents the congestion costs coefficient, reflecting the extra time costs incurred by congestion at ports. Since the congestion at a port depends on the volume of cargo offloaded at the port, the $\beta$s are multiplied by the quantity of cargo transported by respective shipping companies $Q_1$ and $Q_2$. At equilibrium, the shipping costs for shippers of either shipping route would be the same whilst the quantity of cargo distributed amongst them sum up to the total quantity of cargo in the context of our game:

$$\begin{cases} C_1 = C_2 \\ Q_1 + Q_2 = Q \end{cases} \quad (11)$$

Substituting equations 9 and 10 into the equilibrium conditions and solving for the equilibrium, we



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can express the quantity transported by each company per year:

$$Q_1 = \frac{Q}{2\beta} + \frac{1}{2\beta}(\alpha(\frac{d_2}{v_2} - \frac{d_1 - d_s}{v_1} - t_s)) + p_2 - p_1 \tag{12}$$

$$Q_2 = \frac{Q}{2\beta} + \frac{1}{2\beta}(\alpha(\frac{d_1 - d_s}{v_1} + t_s - \frac{d_2}{v_2})) + p_1 - p_2 \tag{13}$$

Given that $T_i$ is the total time a ship from Company i spend sailing per year and $w_i$ is the freight capacity per ship from Company i per trip, we get the expression of the total quantity transported per ship per year for each Company:

$$w_1(\frac{T_1}{t_1}) = \frac{w_1 T_1 v_1 v_s}{v_s(d_1 d_s) + v_1 d_s} \tag{14}$$

$$w_2(\frac{T_2}{t_2}) = \frac{w_2 T_2 v_2}{d_s} \tag{15}$$

Dividing the total quantities transported per year $Q_1$, $Q_2$ by the quantities transported per ship per year, we get the number of ships required for each company:

$$N_1 = \frac{Q_1}{(\frac{w_1 T_1 v_1 v_s}{v_s(d_1 d_s) + v_1 d_s})} = \frac{Q_1[v_s(d_1 - d_s) + v_i d_s]}{w_1 T_1 v_1 d_s} \tag{16}$$

$$N_2 = \frac{Q_2}{\frac{w_2 T_2 v_2}{d_2}} = \frac{Q_2 d_2}{w_2 T_2 v_2} \tag{17}$$

Now we consider the costs for each shipping company. Following past literature, the subsidy granted by the VSRS ($\eta$) is proportional to the fuel consumed by the vessel spends within the SRZ, thus we can subtract it from the fuel price. We follow the proven equation that the consumption of fuel is a cubic term of the vessel speed, hence $f(v) = kv^3$ where $k$ denotes the coefficient of fuel consumption. For the fixed portion of the costs, it comprises of aspects such as the costs of the ship crew, maintenance costs, insurance, and more, which would be all denoted under the coefficient $r_i$. Therefore, we get the following two expressions for each company:

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$$\text{Company 1's variable cost } (C_{11}) = \frac{(p_f - \eta) f(v_s) t_s N_1 T_1 v_1 v_s}{v_s(d_1 - d_s) + v_1 d_s} + \frac{p_f f(v_s)(t_1 - t_s) N_1 T_1 v_1 v_s}{v_s(d_1 - d_s) + v_1 d_s} \tag{18}$$

$$= \frac{Q_1 p_f k_1 (d_1 - d_s) v_1^2}{w_1} + Q_1 (p_f - \eta)\phi_1, \text{ where } \phi_1 = \frac{k_1 d_s v_s^2}{w_1} \tag{19}$$

$$\text{Company 1's fixed cost } (C_{12}) = Q[\frac{r_1(d_1 - d_s)}{w_1 p_1 v_1} + \frac{r_1 d_s}{w_1 p_1 v_s}] \tag{20}$$

$$\text{Company 2's variable cost } (C_{21}) = \frac{p_f f(v_2) t_2 N_2 T_2 v_2}{d_2} \tag{21}$$

$$= \frac{Q_1 p_f k_2 d_2 v_2^2}{w_2} \tag{22}$$

$$\text{Company 2's fixed cost } (C_{22}) = Q(\frac{r_2 d_2}{w_2 T_2 v_2}) \tag{23}$$

Hence, the profit functions of shipping companies 1 and 2 can be represented as follows with $w_{pi}, \forall i \in \{1, 2\}$ representing the port docking fees:

$$\pi_1^s = Q_1(p_1 - w_{p1}) - (C_{11} + C_{12}) \tag{24}$$

$$\pi_2^s = Q_2(p_2 - w_{p2}) - (C_{21} + C_{22}) \tag{25}$$

Now, to find the optimal operating vessel speed, we solve for the partial derivative of the profit function with regards to $p_i$ and $v_i$:

$$\frac{\partial \pi_1^s}{\partial p_1} = (p_1 - w_{p1} - C_1)\frac{\partial Q_1}{\partial p_1} + Q_1 = 0 \tag{26}$$

$$\frac{\partial \pi_1^s}{\partial v_1} = (p_1 - w_{p1} - C_1)\frac{\partial Q_1}{\partial p_1} - Q_1(d_1 - d_s)(\frac{2 p_f k_1 v_1}{w_1} - \frac{r_1}{w_1 p_1 v_1^2}) = 0 \tag{27}$$

Similarly, for shipping company 2:

$$\frac{\partial \pi_2^s}{\partial p_2} = (p_2 - w_{p2} - C_2)\frac{\partial Q_2}{\partial p_2} + Q_2 = 0 \tag{28}$$

$$\frac{\partial \pi_2^s}{\partial v_2} = (p_2 - w_{p2} - C_2)\frac{\partial Q_2}{\partial p_2} - Q_2 d_2(\frac{2 p_f k_2 v_2}{w_2} - \frac{r_2}{w_2 p_2 v_2^2}) = 0 \tag{29}$$

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To verify the concavity of the profit function, we now compute its Hessian matrix.

$$H_{s1} = \begin{bmatrix} \frac{\partial^2 \pi_1^s}{\partial p_1^2} & \frac{\partial^2 \pi_1^s}{\partial p_1 \partial v_1} \\ \frac{\partial^2 \pi_1^s}{\partial v_1 \partial p_1} & \frac{\partial^2 \pi_1^s}{\partial v_1^2} \end{bmatrix} \tag{30}$$

$$= \begin{bmatrix} -\frac{1}{\beta} & -\frac{1}{2\beta}(\frac{p_f k_1 (d_1-d_s) 2v_1}{w_1} - \frac{r_1(d_1-d_s)}{w_1 T_1 v_1^2}) + \frac{\alpha(d_1-d_s)}{2\beta v_1^2} \\ -\frac{1}{2\beta}(\frac{p_f k_1 (d_1-d_s) 2v_1}{w_1} - \frac{r_1(d_1-d_s)}{w_1 T_1 v_1^2}) + \frac{\alpha(d_1-d_s)}{2\beta v_1^2} & \frac{\alpha(d_1-d_s)}{\beta v_1^2} - \frac{2Q_1 \alpha(d_1-d_s)}{v_1^3} - \frac{alpha^2 (d_1-d_s)^2}{v_1^4} - Q_1 \frac{\partial^2 \theta_1}{\partial v_1^2} \end{bmatrix} \tag{31}$$

Since $\det(H_{s1}) > 0$ and $-\frac{1}{\beta} < 0$, we can confirm that this point is indeed a local maxima of the profit function. We can now compute the optimal values for the operating parameters corresponding to this profit value. Solving the first-order PDE of the profit function with respect to $v_1$ and $v_2$ computed previously, we find that the optimal vessel speed for the two companies could be represented by the same general equation:

$$v_i = (\frac{\alpha w_i T_i + r_i}{2 p_f T_i k_i})^{\frac{1}{3}}, \text{ where } i\text{=}1,2 \tag{32}$$

As a result, we can see that implementing a VSRS does not influence the optimal vessel speed $v_i$ since company 1 who implemented it has the identical $v_{Ni}$ expression as company 2 who did not implement it. However, it does impact the optimal shipping price that these shipping companies charge shippers with as we will demonstrate later.

### 3.1 Nash-Nash (N-N) game

We first consider the Nash-Nash scenario where we model both the interaction between the ports and interactions between shipping companies as Nash games. Solving for Equations 26 and 28, we get the expressions for the optimal freight rates:

$$p_{N1} = \beta Q + \frac{1}{3}[2w_{p1} + w_{p2} + 2C_1 + C_2 + \alpha(\frac{d_2}{v_2} - \frac{d_1 - d_s}{v_1} - t_s)] \tag{33}$$

$$p_{N2} = \beta Q + \frac{1}{3}[w_{p1} + 2w_{p2} + C_1 + 2C_2 + \alpha(-\frac{d_2}{v_2} + \frac{d_1 - d_s}{v_1} + t_s)] \tag{34}$$

Here we assume that all vessels used by the two shipping companies are identical, hence $r_1 = r_2 = r$, $w_1 = w_2 = w$, $T_1 = T_2 = T$, $k_1 = k_2 = k$ are all exogenously given. Therefore, the optimal speed $v_N i$ for both companies' ships becomes the same and can now be represented by simply $v$. Substituting these values in, the optimal freight rate $p_{Ni}$ now becomes:

$$p_{N1} = \beta Q + \frac{2w_{Np1} + w_{Np2}}{3} + \frac{d_2}{2v}(\alpha + \frac{r}{wT}) + \frac{(d_1 - d_s)r}{wTv} + \frac{2}{3}(\phi_2 - \eta\phi_1) - \frac{\alpha t_s}{3} \quad (35)$$

$$p_{N2} = \beta Q + \frac{w_{Np1} + 2w_{Np2}}{3} + \frac{d_1 - d_s}{2v}(\alpha + \frac{r}{wT}) + \frac{d_2 r}{wTv} + \frac{1}{3}(\phi_2 - \eta\phi_1 + \alpha t_s) \quad (36)$$

Placing these values for $p_{N1}$ and $p_{N2}$ into the quantity equations, we get:

$$Q_{N1} = \frac{1}{2\beta}(\beta Q + \frac{w_{Np2} - w_{Np1}}{3} + \phi_3 + \frac{\eta\phi_1}{3} \quad (37)$$

$$Q_{N2} = \frac{1}{2\beta}(\beta Q + \frac{w_{Np1} - w_{Np2}}{3} - \phi_3 - \frac{\eta\phi_1}{3} \quad (38)$$

Given these optimal values of the shipping companies, we input them into the profit function of the port to obtain the optimal docking fees the ports should charge given that shipping companies achieve their optimal profits. The profit function of port 1 and 2 are shown below, placing the optimal quantities $Q_{N1}$ and $Q_{N2}$ into the function, we can find the optimal docking fee $w_{Np1}$ and $w_{Np2}$.

$$\max_{w_{p1}} \pi_1^P = (w_{p1} - c)Q_1 - \eta Q_1 \phi_1 \quad (39)$$

$$\max_{w_{p2}} \pi_2^P = (w_{p2} - c)Q_2 \quad (40)$$

$$w_{Np1} = 3\beta Q + c + \phi_3 + \eta\phi_1 \quad (41)$$

$$w_{Np2} = 3\beta Q + c - \phi_3 \quad (42)$$

Substituting equations (41) and (42) into (37) and (38), we can simplify the optimal quantities as such:

$$Q_{N1} = \frac{1}{2\beta}(\beta Q + \frac{\phi_3}{3}) \quad (43)$$

$$Q_{N2} = \frac{1}{2\beta}(\beta Q - \frac{\phi_3}{3}) \quad (44)$$



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And substituting equations (41) and (42) into (35) and (36), the optimal shipping prices becomes:

$$p_{N1} = 4\beta Q + c + \frac{2d_2}{3v}(\alpha + \frac{r}{wT}) + \frac{d_1 - d_s}{6v}(\frac{5r}{wT} - \alpha) + \frac{1}{9}(5\phi_2 - 4\alpha t_s) \tag{45}$$

$$p_{N2} = 4\beta Q + c + \frac{2(d_1 - d_s)}{3v}(\alpha + \frac{r}{wT}) + \frac{d_2}{6v}(\frac{5r}{wT} - \alpha) + \frac{4}{9}(\phi_2 + \alpha t_s) \tag{46}$$

### 3.2 Nash-Stackelberg (N-S) game

Now we move onto the N-S scenario, where shipping company 1 is the Stackelberg leader and company 2 is the follower. Using a backward induction procedure, we first compute the best response function of company 2, i.e. its optimal shipping price as it takes place in the latter step in this sequential game. By extracting $p_2$ from equation (15), we get:

$$p_{S2} = \frac{1}{2}[p_{S1} + w_{Sp2} + \beta Q + \frac{d_2}{2v}(\frac{r}{wT} - \alpha) + \alpha(\frac{d_1 - d_s}{v}) + t_s] \tag{47}$$

Placing this expression into the profit function of shipping company 1 or equation (12), we get that:

$$p_{S1} = \frac{1}{2}[w_{Sp1} + w_{Sp2} + 3\beta Q + \frac{\alpha(3d_2 - d_1 + d_s)}{2v} + \frac{3r(d_2 + d_1 - d_s)}{2wTv} + \phi_2 - \eta\phi_1 - \alpha t_s] \tag{48}$$

Putting the $p_{S1}$ into equation (14) we can get the optimal pricing for company 2, hence how company 2 will respond to the decision made by company 1:

$$p_{S2} = \frac{1}{4}[w_{Sp1} + 3w_{Sp2} + 5\beta Q + \frac{3r(3d_2 + d_1 - d_s)}{2wTv} + \frac{\alpha(3d_1 - 3d_s + d_2)}{2v} + \phi_2 - \eta\phi_1 + \alpha t_s] \tag{49}$$

Now substituting the optimal prices $p_{S1}$ and $p_{S2}$ into the quantity expressions, we can get:

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$$Q_{S1} = \frac{1}{8\beta}(3\beta Q + w_{Sp2} - w_{Sp1} + 3\phi_3 + \eta\phi_1) \tag{50}$$

$$Q_{S2} = \frac{1}{8\beta}(5\beta Q - w_{Sp2} + w_{Sp1} - 3\phi_3 - \eta\phi_1) \tag{51}$$

Using this we now compute the optimal docking fees similarly as in the N-N case by inputting the optimal quantity into the profit function of the ports:

$$w_{Sp1} = \frac{11}{3}\beta Q + c + \phi_3 + \eta\phi_1 \tag{52}$$

$$w_{Sp2} = \frac{13}{3}\beta Q + c - \phi_3 \tag{53}$$

Substituting equations 52 and 53 back into the optimal quantity expressions 54 and 55, we can now find the final expression of the optimal quantities:

$$Q_{S1} = \frac{1}{8\beta}(\frac{11}{3}\beta Q + \phi_3) \tag{54}$$

$$Q_{S2} = \frac{1}{8\beta}(\frac{13}{3}\beta Q - \phi_3) \tag{55}$$

And now finally, we can get the final expression for the optimal prices

$$p_{S1} = \frac{1}{2}[11\beta Q + 2c + \frac{\alpha(3d_2 - d_1 + d_s)}{2v} + \frac{3r(d_2 + d_1 - d_s)}{2wTv} + \phi_2 - \alpha t_s] \tag{56}$$

$$p_{S2} = \frac{1}{4}[\frac{65}{3}\beta Q + 4c + \frac{\alpha(5d_1 - 5d_s - d_2)}{2v} + \frac{r(7d_2 + 5d_1 - 5d_s)}{2wTv} + \frac{5(\phi_2 + \alpha t_s)}{3}] \tag{57}$$

### 3.3 The impact of VSRS on agents' operations

Now we can analyze how the VSRS can impact the operational variables of the ports and the shipping companies. Here we represent the difference in costs per unit time between shipping company 1 choosing to adhere to the speed limit and not adhering to it. Hence, it could also be seen as the opportunity cost of slowing down below the vessel speed limit:



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$$\Delta C_s = p_f k(v_s^2 - v^2) + (\alpha w + \frac{r}{T})(\frac{1}{v_s} - \frac{1}{v}) \tag{58}$$

The first group of terms $Fk(v_s^2 - v^2)$ represents the difference in fuel costs per unit of time. The second group $(\alpha w + \frac{r}{T})(\frac{1}{v_s} - \frac{1}{v})$ refers to the sum of the variable transporting costs $(\alpha w)$ and the fixed costs $(\frac{r}{T})$, which are both time-dependent, hence why $(\alpha w + \frac{r}{T})$ is multiplied by the difference in time $(\frac{1}{v_s} - \frac{1}{v})$. As we are only considering the distance within the SRZ, we can omit the $d_s$ in the equation and replace it with 1 as seen in the last bracket of the equation above. By substituting equation 32 into the above equation, it could be represented as follows:

$$\Delta C_s = p_f k v_s^2 + \frac{1}{v_s}(\alpha w + \frac{r}{T}) - \frac{3}{2v}(\alpha w + \frac{r}{T}) \tag{59}$$

#### 3.3.1 Ports

From Equations 41 and 52, taking the derivative of the port 1's service charges with respect to the distance traveled in the SRZ, we find that the derivative in both N-N and N-S scenarios is equal:

$$\frac{\partial w_{Np1}}{\partial d_s} = \frac{\partial w_{Sp1}}{\partial d_s} = \frac{1}{2wv}(\alpha w + \frac{r}{T}) - \frac{1}{3w}[p_f k v_s^2 + \frac{1}{v_s}(\alpha w + \frac{r}{T})] + \frac{\eta k v_s^2}{w} \tag{60}$$

$$= \frac{1}{w}(\eta k v_s^2 - \frac{1}{3}\Delta C_s) \tag{61}$$

$$\frac{\partial w_{Np1}}{\partial d_s} = \frac{\partial w_{Sp1}}{\partial d_s} = \frac{1}{w}(\eta k v_s^2 - \frac{1}{3}\Delta C_s) \tag{62}$$

By having the exact same derivative under both scenarios, we can arrive at the following proposition:

**Proposition 2** *: Implementing VSRS with the same parameter under different competition structures between shipping companies results in changes of the same magnitudes in their corresponding port's service charges.*

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Interpreting Equation 62, we can obtain the following conclusions for port 1 under both scenarios:

1. When $\Delta C_s > 3\eta k v_s^2$, Port 1's service charges decrease as $d_s$ increases

2. When $\Delta C_s < 3\eta k v_s^2$, Port 1's service charges increase as $d_s$ increases

Using the same procedure as port 1 above but with Equations 42 and 53, we can get the following for port 2:

$$\frac{\partial w_{Np2}}{\partial d_s} = \frac{\partial w_{Sp2}}{\partial d_s} = \frac{1}{3w}[p_f k v_s^2 + \frac{1}{v_s}(\alpha w + \frac{r}{T}) - \frac{1}{2wv}(\alpha w + \frac{r}{T})] \tag{63}$$

$$= \frac{1}{3w}\Delta C_s \tag{64}$$

$$\frac{\partial w_{Np2}}{\partial d_s} = \frac{\partial w_{Sp2}}{\partial d_s} = \frac{1}{3w}\Delta C_s \tag{65}$$

From Equation 65, we can similarly infer the following conclusions for Port 2:

1. When $\Delta C_s > 0$, Port 2's service charges increase as $d_s$ increases

2. When $\Delta C_s < 0$, Port 2's service charges decrease as $d_s$ increases

Overall, when the opportunity cost of reducing vessel speed $\Delta C_s$ is greater than 3 times the subsidy granted per unit distance traveled in the SRZ, port 1's service charges decrease, hence allowing lower freight rates for the corresponding shipping company 1, attracting higher demand and thus ultimately increasing the profits of shipping line 1. Conversely, when it is less than the same value, its service charges increase, leading to lowered profits for shipping line 1. For Port 2, when $\Delta C_s$ is positive, its service charges increase as $d_s$ increases, and vice versa when $\Delta C_s$ is positive. There could also exist an intermediary case when $0 < \Delta C_s < 3\eta k v_s^2$, in which both ports' service charges would increase from an increase in $d_s$. Since we assume market demand $Q$ to be constant, this would not lead to a decrease in both the corresponding shipping companies' profits. Hence, between these value boundaries, the flow of demand will be determined by the difference in each port's derivative value $\frac{\partial w_{jpi}}{\partial d_s}$ where the port with the larger derivative value would lose demand while



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the other port gains the same amount as given by 11. On the other hand when $3\eta k v_s^2 < \Delta C_s < 0$, VSRS acts to decrease service charges for both parties. These intermediary cases will be further studied in the numerical analysis in Section 3.5.

### 3.3.2 Shipping companies

Moving on to the freight rate aspect of this problem. Taking the derivative of the optimal freight rates $p_{N1}$ and $p_{N2}$ with respect to $d_s$ under both N-N and N-S scenarios, we see that, unlike the port service charges, the derivative for the freight rates against $d_s$ is different under the two scenarios. For the N-N scenario, we get:

$$\frac{\partial p_{N1}}{\partial d_s} = \frac{5}{9w}\Delta C_s + \alpha(\frac{1}{v} - \frac{1}{v_s}) \tag{66}$$

$$\frac{\partial p_{N2}}{\partial d_s} = \frac{4}{9w}\Delta C_s \tag{67}$$

Interpreting the equations, we yield the following conclusions for shipping companies' freight rates under the N-N scenario:

1. When $\Delta C_s > -\frac{9w}{5}\alpha(\frac{1}{v} - \frac{1}{v_s})$, shipping company 1's freight rate decrease as $d_s$ increases

2. When $\Delta C_s < -\frac{9w}{5}\alpha(\frac{1}{v} - \frac{1}{v_s})$, shipping company 1's freight rate increase as $d_s$ increases

3. When $\Delta C_s > 0$, shipping company 2's freight rate increases as $d_s$ increases

4. When $\Delta C_s < 0$, shipping company 2's freight rate decreases as $d_s$ increases

For freight rates of shipping companies under the N-S scenario, we can obtain the following derivatives and corresponding conclusions

$$\frac{\partial p_{S1}}{\partial d_s} = \frac{1}{9w}\Delta C_s + \alpha(\frac{1}{v} - \frac{1}{v_s}) \tag{68}$$

$$\frac{\partial p_{S2}}{\partial d_s} = \frac{4}{9w}\Delta C_s \tag{69}$$



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1. When $\Delta C_s > 2\alpha w(\frac{1}{v_s} - \frac{1}{v})$, shipping company 1's freight rate decrease as $d_s$ increases

2. When $\Delta C_s < 2\alpha w(\frac{1}{vs} - \frac{1}{v})$, shipping company 1's freight rate increase as $d_s$ increases

3. When $\Delta C_s > 0$, shipping company 2's freight rate increases as $d_s$ increases

4. When $\Delta C_s < 0$, shipping company 2's freight rate decreases as $d_s$ increases

As shown above, the derivative of the shipping companies' freight rates are different under the two scenarios, allowing us to reach the following proposition:

**Proposition 3** *: Implementing VSRS with the same parameter under different competition structures between shipping companies results in changes of different magnitudes in their freight rates*

Similar to the aforementioned port service charges, there may exist intermediary value boundaries where when $0 < \Delta C_s < -\frac{9w}{5}\alpha(\frac{1}{v} - \frac{1}{v_s})$ for the N-N case and when $0 < \Delta C_s < 2\alpha w(\frac{1}{vs} - \frac{1}{v})$ for the N-S case, implementing VSRS simultaneously incur freight rate increases or decreases for both shipping companies. Section 3.5 will conduct a numerical analysis of this.

Overall, from the above analysis under both scenarios, we can arrive at a general proposition as follows:

**Proposition 4** *: Implementing VSRS can cause price increases or decreases on one or both shipping lines depending on the choice of the vessel speed limit $v_s$.*

While this difference in result is shown to be dependent on the choice of the speed limit $v_s$, the magnitude of these gains and losses is dependent on the changes in the radius $d_s$ of the SRZ. The boundaries determined by $v_s$ and the magnitude of the effect of $d_s$ are also shown to be different under different competition structures. Depending on these boundaries determined by $v_s$, implementing VSRS can either produce the same price impact on both shipping lines or an opposite effect, meaning that one shipping line grows in profit at the expense of incurring a loss on the other. These boundaries and magnitudes will be studied numerically in the numerical analysis in Section 3.5.



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### 3.4 The impact of VSRS on emissions

Moving on from the economic impacts of implementing VSRSs, we now focus on the environmental side of this problem. Let the baseline case be when port-1 does not implement the VSRS and vessels of shipping company 1 do not reduce their speed. Without loss of generality, we examine only the sulfur emissions produced. The following analysis could also be done for CO2 and other types of emissions by simply a change in one variable's value. We represent the $SO_2$ emissions for shipping company 1 and 2 in this case as $E_{j01}$ and $E_{j02}$, $\forall j \in \{n, s\}$ where the 0 in the subscript represents that no VSRS is implemented. Let $S_{content}$ be the percentage amount of sulfur contained per unit mass of fuel. Assuming both companies use the same heavy fuel oil (HFO), 2% of sulfur is released when the fuel is combusted. Hence we can represent the sulfur dioxide emissions as:

$$E_{j01} = \frac{0.02 S_{content} k d_s v^2 Q_{j01}}{w} \tag{70}$$

$$E_{j02} = \frac{0.02 S_{content} k d_s v^2 Q_{j02}}{w} \tag{71}$$

When VSRS is implemented, the emissions expressions become:

$$E_{j1} = \frac{0.02 S_{content} k d_s v_s^2 Q_{j1}}{w} \tag{72}$$

$$E_{j2} = \frac{0.02 S_{content} k d_s v_s^2 Q_{j2}}{w} \tag{73}$$

Let $\Delta E_{N1}$ be the difference between the emission produced when VSRS is implemented and when it is not implemented.

$$\Delta E_{N1} = E_{N1} - E_{N01} \tag{74}$$

$$= \frac{0.02 S_{content} k d_s}{w} (v_s^2 Q_{N1} - v^2 Q_{N01}) \tag{75}$$

As concluded previously when $\Delta C_s \geq 0$, $Q_{n1} \leq Q_{n01}$ and $v_s \leq v$. Therefore, in this situation, $\Delta E_{n1}$ would be negative, hence the implementation of VSRS would have its intended reduction effect



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on $SO_2$ emissions. For analyzing the cases when $\Delta C_s < 0$, we first substitute the full expressions of $Q_{n1}$ and $Q_{n01}$ into 75.

$$\Delta E_{N1} = \frac{0.01 S_{content} k d_s}{w\beta}[\beta Q(v_s^2 - v^2) + \frac{1}{3}(v_s^2\phi_3 - v^2\phi_4)] \tag{76}$$

$$= \frac{0.01 S_{content} k d_s}{6w\beta}[3\beta Q(v_s^2 - v^2) + (v_s^2\phi_3 - v^2\phi_4)] \tag{77}$$

We first set $\Delta E_{N1} < 0$ to find the corresponding conditions that satisfy this result.

$$3\beta Q(v_s^2 - v^2) < (v^2\phi_4 - v_s^2\phi_3) \tag{78}$$

$$(3\beta Q + \phi_3)(v_s^2 - v^2) < v^2(\phi_4 - \phi_3) \tag{79}$$

$$(3\beta Q + \phi_3)(v_s^2 - v^2) < \frac{d_s \Delta C_s v^2}{3w} \tag{80}$$

Since $\Delta C_s < 0$, $(3\beta Q+\phi_3)(v_s^2-v^2)$ would also be negative. Furthermore due to $v_s \le v$, $3\beta Q+\phi_3$ would be non-negative. Hence, when $\Delta C_s < 0$, for VSRS to have a positive effect on $SO_2$ emissions, $3\beta Q + \phi_3$ must be positive so that $\Delta E_{N1} < 0$. Then similarly, when $3\beta Q + \phi_3 < 0$, $\Delta E_{N1} \ge 0$, thus VSRS would have an unintended negative effect on $SO_2$ emissions.

For the N-S scenario, following the same steps, we similarly find that an increase in the $d_s$ does not guarantee a decrease in port service charges. When $\Delta C_s$ is positive, port 1's emission will decrease while that of port 2 will increase accordingly. When $\Delta C_s$ is negative, changes to port 1's emissions would then be determined by other similar conditions that could be found following the same analysis procedure as the N-N case. Both the N-N and N-S scenarios' emission will be further studied in detail in the numerical analysis of the following section.

## 3.5 Numerical analysis

Firstly, the price of heavy fuel oil (HFO), which most containerships use, as of December 2023, is at \$428/ton. As for containership sizes, the carrying capacity can vary from a few hundred TEUs to 20000+ TEUs (the MSC Irina). Following past literature, we consider the most popular of the fleet, the Post Panamax which has an approximate capacity $w$ of 6000 TEU. Forecasted by the Institute

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of Water Resources, Post Panamax vessels are projected to account for 62% of total fleet capacity in the world by 2030 [31]. From data provided by the IMO [19], the average time in transit per year for a vessel is approximately $T = 6480$ hours. Without loss of generality, we set both $d1$ and $d2$ to be 2000 for a fairer comparison. Since the daily operating cost ($r$ in our case) varies from \$8150 for a 4000 TEU Panamax vessel to \$11575 for a 10000 TEU Mega-Post Panamax vessel, we assume this value to be \$10000 for our case, thus an annual fixed cost of $r = \$2.7$ million. The average cargo value is estimated at \$10957/TEU, meaning an in-transit inventory cost $a = \$0.87/\text{h/TEU}$ given that capital costs = 7% [32]. Table 3 summarizes the parameter values used in this numerical analysis.

#### 3.5.1 Freight rates

We first analyze how implementation details of VSRS may affect shipping company operations, hence how the vessel speed limit $v_s$, and the SRZ radius $d_s$ affect the optimal operating parameters of shipping companies. Each graph includes the results for shipping companies 1 and 2 under both N-N and N-S scenarios, hence 4 surfaces would be seen.

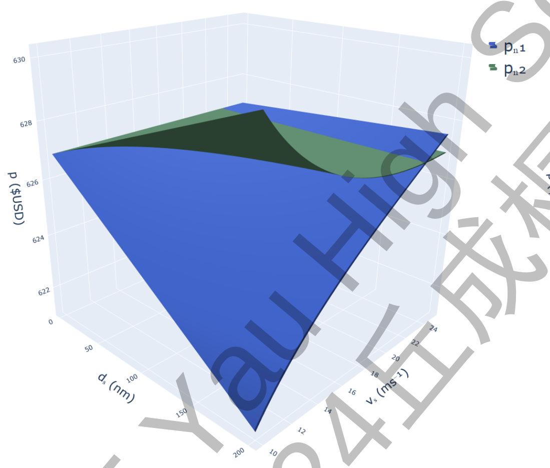


Figure 12: $d_s$ and $v_s$'s relationship with the optimal port freight rates $p$ under the N-N scenario

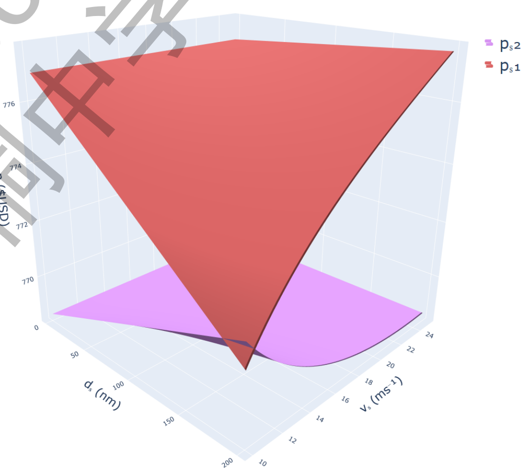


Figure 13: $d_s$ and $v_s$'s relationship with the optimal port freight rates $p$ under the N-S scenario

Figures 12 and 13 display the two variables' relationship with the optimal freight rates. As seen from the distinct difference in value between the N-N and N-S scenarios (one in the low-600 ranges while the other in the high-700 ranges), we can infer the drastic impact that the competition

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structure of shipping companies has on freight rate levels. Expectedly, the presence of a Stackelberg leader in the N-S case displays a larger price difference between the shipping companies due to the leader's pricing power and the sequential nature of the game.

Examining $d_s$ the radius of the SRZ, we can see when $v_s$ is set at a higher level, $d_s$ essentially does not affect the freight rates. However, for a lower vessel speed limit $v_s$, an increase in $d_s$ would increase the freight rates of the shipping company 2s and decrease the rates of the shipping company 1s in both scenarios.

Viewing from the perspective of $v_s$, given a relatively high $d_s$, an increase in $v_s$ would cause an increase in shipping company 1's freight rates and a decrease in that of shipping company 2's regardless of the competition structure. However, as $d_s$ decreases, this effect gradually vanishes as seen by the surface becoming flat towards the left.

Examining the impact of $d_s$ on the freight rates, we see that for both scenarios, when $v_s$ is set relatively low, the freight rate of shipping company 1 decreases as $d_s$ increases and that of shipping company 2 increases as $d_s$ increases. However, when $v_s$ is raised to a relatively high level, the opposite effect occurs. Hence, by fitting in numerical values, we have demonstrated Proposition 4 of how $v_s$ plays a determining role in deciding the impact of VSRS on shipping lines' prices. To determine the exact boundary values, we can solve for $\Delta C_s$ when it equals each of the boundary equations. For the case of shipping company 1's freight rates, we can see that in both N-N and N-S scenarios, the boundary value equation contains a term that is the difference between $\frac{1}{v}$ and $\frac{1}{v_s}$. Therefore, the boundary value $v_s$ for both shipping company 1 occurs at when it is set equal to the optimal vessel speed $v$. To find this boundary value for shipping company 2, we can substitute the numerical values to find $v_s$ when $\Delta C_s = 0$ for both scenarios.

We find that this value in both scenarios is $23.0(3s.f.)$, the same exact value we have computed for shipping company 1. Hence, we can conclude that by implementing VSRS with $v_s$ set at any value lower than the optimal vessel speed $v$ in the shipping companies' self-interest would cause a reduction in a port's freight and rate and an increase in the competing port's freight rate. As seen from the slope of the surfaces in Figure 14 and 15, the freight rates of shipping company 1 under both scenarios decrease at a much faster rate than that of shipping company 2 increases as $d_s$ increases. This leads us to the following proposition:

**Proposition 5** *: Implementing VSRS (hence an increase in $d_s$) with $v_s$ below the optimal vessel*

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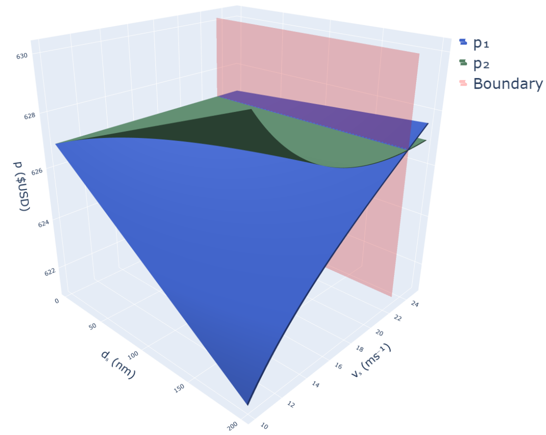


Figure 14: The boundary value of $v_s$ determining the effects of VSRS under the N-N scenario

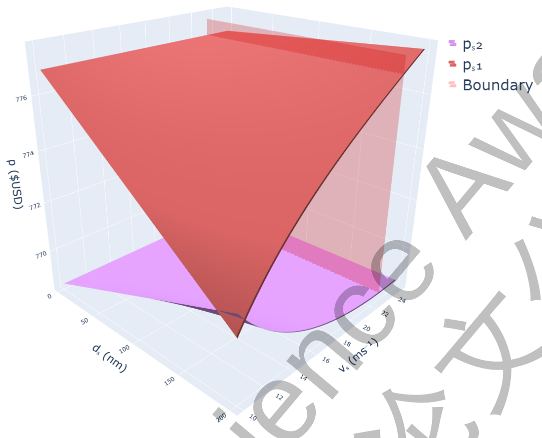


Figure 15: The boundary value of $v_s$ determining the effects of VSRS under the N-S scenario

*speed for shipping companies leads to a decrease in shipping company 1's freight rates due to lower costs, hence taking from the demand of shipping company 1 thus increasing its profits.*

### 3.5.2 Port service charges

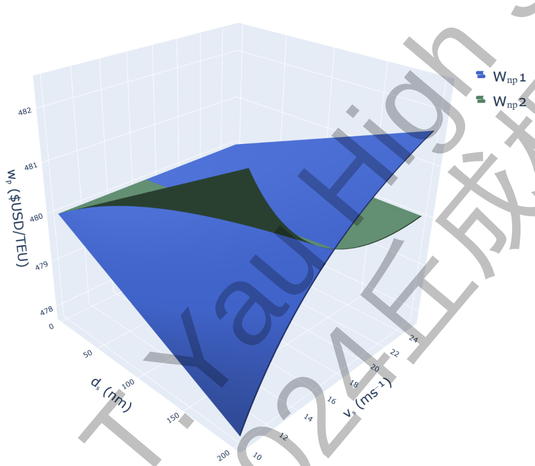


Figure 16: $d_s$ and $v_s$'s relationship with the optimal port freight rates $w_p$ under the N-N scenario

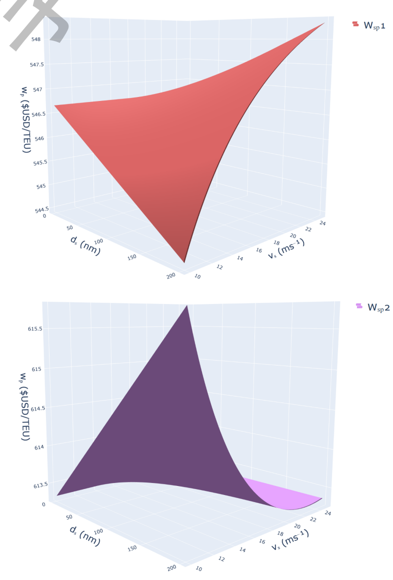


Figure 17: $d_s$ and $v_s$'s relationship with the optimal port service charges $w_p$ under the N-S scenario

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Figure 16 and 17 display the two variables' relationship with the optimal port service charges. The figures of the N-S scenario are split into two for better visual clarity since the two surfaces are originally extremely far apart, making changes on the surface unclear. Different from freight rates, under the N-S scenario, port 1's service charges are lower than that of port 2's. In this case, it is actually well below the service charges of port 2 by over USD 60/TEU. Under the N-N case, the results display apparent similarity with that of the freight rates except that the changes are more exaggerated. However, when we compute the boundary values of $v_s$ in this case, we find that there are two numerically different boundary points as opposed to only one in the previous case for freight rates. By solving for $v_s$ when $\Delta C_s = 3\eta k v_s^2$ (N-N scenario) and when $\Delta C_s = 0$, we find the stationary point on each port service charge's derivative against $d_s$. Using the aforementioned numeric values, we find that only when $v_s$ is set below 19.1($3s.f.$) would increasing $d_s$ lead to a decrease in $w_{p1}$.

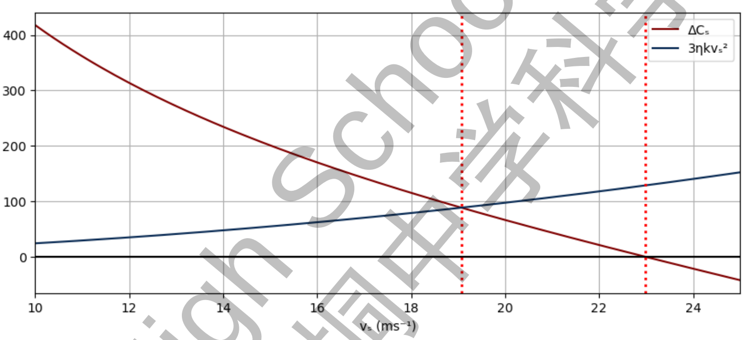


Figure 18: Finding stationary points of port service charges

This would then imply that there indeed exists an intermediary value where $0 < \Delta C_s < 3\eta k v_s^2$ that an increase in $d_s$ causes both port's service charges to increase. Through Figure 18, we can also disprove the existence of a case where $3\eta k v_s^2 < \Delta C_s < 0$, hence it is impossible for both ports' service charges to decrease simultaneously.

#### 3.5.3 Profits

For the N-S, we raised the surface of shipping company 2's profits by 22 million USD for better visualization. Through Figures 19 and 20, we can first see the vast difference in annual profits that different competition structures cause, especially for shipping company 1. Under the N-S scenario,

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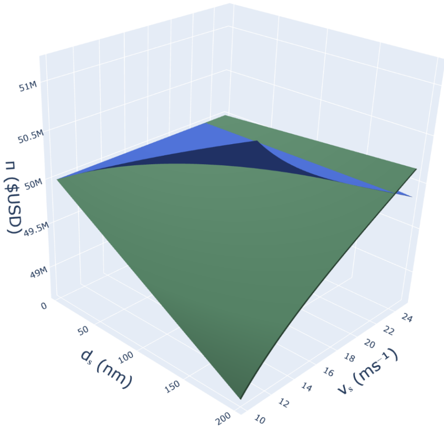


Figure 19: $d_s$ and $v_s$'s relationship with the maximum annual profit achievable $\pi_{ji}$ under the N-N scenario

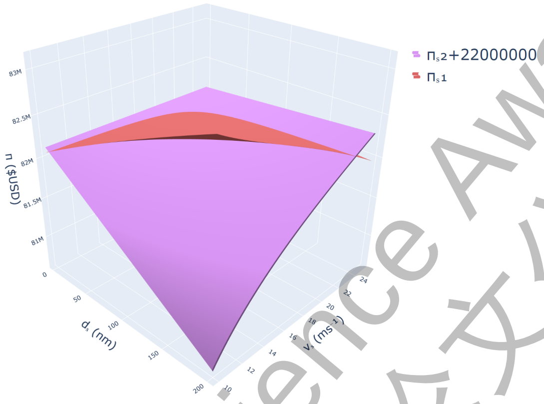


Figure 20: $d_s$ and $v_s$'s relationship with the maximum annual profit achievable $\pi_{ji}$ under the N-S scenario

the profits of shipping companies are much higher than that of the N-N scenario. Moreover, the profit gap between shipping companies 1 and 2 is much more exaggerated in the case of the N-S game, where the profits of company 2 had to be raised by 22 million to achieve a similar level as company 1.

For the implementation of VSRS's impact on their profits, we can see that for a low value of $v_s$, as $d_s$ increases, the profits of shipping company 1 increase and that of shipping company 2 decrease. Hence, this is consistent with the opposite results found for freight rates since a higher freight rate would decrease the demand for the proportionally more, thus producing lower profits. Hence, implementing a VSRS is demonstrated to increase a shipping company's profits while reducing the profits of the competing shipping company, making it a viable strategy under competitive shipping markets all the while reducing environmental damage.

#### 3.5.4 Emissions

Figure 21 displays the two variables' relationship with the minimum achievable emission amount per TEU. For better interpretation, we decide to graph all the results under both scenarios into one. From the graph, as the SRZ radius $d_s$ increases, the emission of shipping companies increases linearly, however at different rates. Port 1's emission under both increases less than that of Port 2's as $d_s$ increases. This trend is especially prominent when $v_s$ is set to a low value. Comparing the

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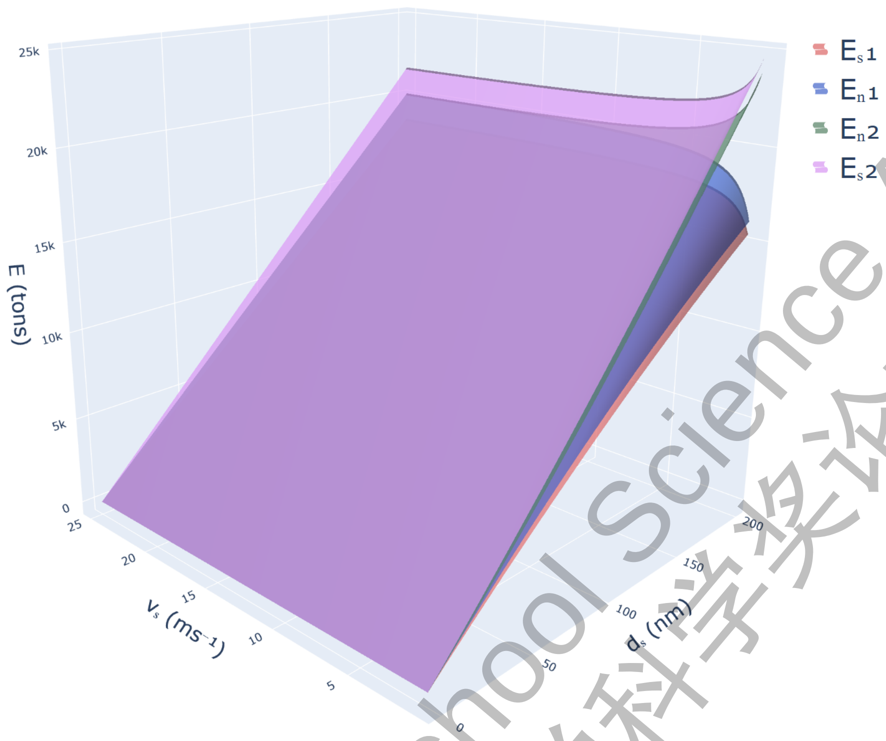


Figure 21: $d_s$ and $v_s$'s relationship with the minimum emission achievable $E$

two scenarios, we see that VSRS achieves a much better result under the N-S scenario as port 1's emissions (red) are drastically lower than that of port 2 (purple). In the N-N scenario, port 1 still achieves a lower emission, however this difference is only noticeable given lower values of $v_s$.

One to note is that as $v_s$ increases, the emission at port 1 under both scenarios increases. This could be attributed to the fact that a higher speed limit decreases the transit time for shipping company 1, hence allowing them to transport a higher volume of cargo annually, leading to higher emissions at their corresponding port. Since shipping company 1 still receives the same amount of subsidy, its operating costs are lower than that of company 2, hence they are more price competitive, leading to higher demand from shippers. From the assumption of constant market demand in Equation 11, this would lead to an equivalent decrease in the demand for shipping company 2, hence reducing the emissions at port 2.

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# 4 Considering the government

Extending from the model established in 3, in this section, we consider the addition of the government into the model from the previous section. We will only consider the N-N scenario for this section. Specifically, the government intervenes by paying for a portion of the subsidy given out by the port implementing VSRS. For consistency, we follow Section 3 and again assume that $r_1 = r_2 = r$, $w_1 = w_2 = w$, $T_1 = T_2 = T$, $k_1 = k_2 = k$ are all exogenously given.

## 4.1 Subsidy-sharing policy

By implementing the subsidy-sharing policy, the government chooses a parameter $z \in [0, 1]$ which represents the percentage of the port's spending on subsidy that the government would cover. Following the same procedure as in Section 3, assuming that the marginal cost of both ports is the same, hence $C_{p1} = C_{p2} = c$, We get the profit functions of the two ports as follows:

$$\pi_1^P = Q_1(w_{p1} - c) - \frac{(1-z)\eta Q_1 k_1 d_s v_s^2}{w_1} \tag{81}$$

$$\pi_2^P = Q_2(w_{p2} - c) \tag{82}$$

As for the government's costs, since we only consider the emissions produced by the port in this policy, we disregard the emissions produced by the vessels traveling outside of the port's SRZ. When vessels are at the port, they do not use their main engine to sail along the straits but their auxiliary engines instead. Hence we denote the fuel consumption during docking as $F_{di} = \frac{Q_i t_{di} f_{di}}{w}$ where "$d$" represents "docking" and $f_{di}$ represents the fuel consumption per unit time when docking. Taking from previous sections, the fuel consumption within the SRZ for each company would be $F_{s1} = \frac{Q_1 k_1 d_s v_s^2}{w_1}$ and $F_{s2} = \frac{Q_2 k_2 d_s v_2^2}{w_2}$ respectively. Therefore, we represent the total annual pollution costs incurred on the government as $\sum_{i=1}^{2} \lambda(F_{si} + F_{di})$, where $\lambda$ is the pollution coefficient, representing the emission costs incurred per ton of fuel combusted. Hence, the objective function of the government in this scenario is:

$$\min C_G = \sum_{i=1}^{2} \lambda(F_{si} + F_{di}) + z\eta F_{s1},\ z \in [0, 1]$$



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The following set of equations showcases the entire game mathematically:

$$\min_z C_G = \sum_{i=1}^{2} \lambda(F_{si} + F_{di}) + z\eta F_{s1},\, z \in [0,\, 1]$$

$$s.t. \begin{cases} \max\limits_{w_{p1}} \pi_1^P = Q_1(w_{p1} - c) - (1-z)\eta F_{s1} \\ \max\limits_{w_{p2}} \pi_2^P = Q_2(w_{p2} - c) \\ s.t. \begin{cases} \max\limits_{p_1,\, v_1} \pi_1^S = Q_1(p_1 - w_{p1}) - (C_{11} + C_{12}) \\ \max\limits_{p_2,\, v_2} \pi_2^S = Q_2(p_2 - w_{p2}) - (C_{21} + C_{22}) \\ s.t. \left\{ \; p_i \in \arg\max \pi_i^S,\, i \in \{1,\, 2\} \right. \end{cases} \end{cases}$$

## 4.2 Results of subsidy-sharing model

Following the same procedure in Section 3 and assuming that $F_{d1} = F_{d2} = F_d$ thus $t_{d1} = t_{d2} = t_d$ and $f_{d1} = f_{d2} = f_d$ to be exogenously given, the optimal value of $z$ computed, denoted as $z^*$, is shown below:

$$z^* = \frac{\lambda(\theta_7 - \theta_6) - 9\beta Q - 3\theta 5}{2\eta\theta_4}, \text{ where } \theta_6 = \frac{kd_s v_s^2 + t_d f_d}{w} \text{ and } \theta_7 = \frac{kd_s v^2 + t_d f_d}{w} \quad (83)$$

We now analyze how this optimum would change with changes in the operational parameters of the ports and shipping companies. In the first case where $0 < \lambda(\theta_7 - \theta_6) - 9\beta Q - 3\theta_5 < 2\eta\theta_4$, $z^* = \frac{\lambda(\theta_7-\theta_6)-9\beta Q-3\theta_5}{2\eta\theta_4}$. As seen from the derivative $\frac{\partial z^*}{\partial d_s} = \frac{9\beta Q w}{2\eta k d_s^2 v_s^2} + \frac{d_2-d_1}{4\eta k d_s^2 v_s^2 v}(\alpha w + \frac{r}{T})$, when $d_2 > d_1$, the optimal value of $z^*$ increases as $d_s$ increases. In the other case where $d_2 < d_1$, $z^*$ will increase as $d_s$ increases if $18\beta Q w v > (d_1 - d_2)(\alpha w + \frac{r}{T})$ and decrease if $18\beta Q w v < (d_1 - d_2)(\alpha w + \frac{r}{T})$. Substituting in the previous numeric values, we find that is it impossible for the case where $z^*$ decreases as $d_s$ increases since that would require the difference between the two shipping routes $d_1$ and $d_2$ to be larger than $44000(3s.f.)$ nautical miles which is over twice the circumference of the Earth. This then affirms that the sharing percentage $z$ would always increase as port 1 increase their SRZ diameter $d_s$.

**Proposition 6** *: By receiving higher subsidies, port 1 should then be able to lower their service charges $w_{p1}$ due to lowered operating costs.*



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$$\frac{\partial w_{p1}}{\partial d_s} = -\frac{2}{3w}\Delta C_s - \frac{\lambda k(v^2 - v_s^2)}{3w} + \frac{\eta k v_s^2}{w} \tag{84}$$

We can now prove Proposition 6 by examining how $w_{p1}$ changes with $d_s$ under this subsidy-sharing policy. Firstly, for Port 1, the derivative $\frac{\partial w_{p1}}{\partial d_s} = -\frac{2}{3w}[p_f k v_s^2 + \frac{1}{v_s}(\alpha w + \frac{r}{T}) - \frac{3}{2v}(\alpha w + \frac{r}{T})] - \frac{\lambda k(v^2-v_s^2)}{3w} + \frac{\eta k v_s^2}{w} = -\frac{2}{3w}\Delta C_s - \frac{\lambda k(v^2-v_s^2)}{3w} + \frac{\eta k v_s^2}{w}$ suggests that when $\Delta C_s > \frac{3\eta k v_s^2 - \lambda k(v^2-v_s^2)}{2}$, the port service fee $w_{p1}$ would decrease as $d_s$ increases and vice versa. Now, using previous numerical values, we can compute exactly what these boundaries are. Since the amount of $CO_2$ produced per ton of heavy fuel oil combusted is around 4.5 tons [33] and the social cost of carbon (SCC) is estimated at 51USD per ton [34], $\lambda$ will take the value of 229.5. Substituting in the values, we can see that as long as $v_s$ is set below 20.8 ($3s.f.$), the derivative would always be negative. We can hence arrive at the following corollary corresponding to Proposition 6.

**Corollary 1** *: Increasing $d_s$ with a speed limit $v_s$ set lower than $20.8m/s$ would decrease the port service charges $w_{p1}$.*

Compared to the case in Section 3, this boundary value of $20.8m/s$ is slightly higher than that with government intervention of $19.1m/s$. Additionally, plotting the PDEs as $v_s$ changes (Figure 22), we observe that the gradient in the subsidy-sharing case decreases much faster than in the original case, suggesting that increases in $d_s$ would cause an increasingly larger decrease in $w_{p1}$ as $v_s$ decreases. Therefore, implementing a subsidy-sharing policy would benefit port 1 by making it easier for them to lower their freight rates.

On the other hand, for port 2, we find that the derivative $\frac{\partial w_{p2}}{\partial d_s} = \frac{\Delta C_s - \lambda k(v^2-v_s^2)}{6w}$ suggests that when $\Delta C_s > \lambda k(v^2 - v_s^2)$, $w_{pi}$ would increase as $d_s$ increases. Similarly, substituting the numerical values in, we find that as long as $v_s$ is set below a value of 23.0 ($3s.f.$), $w_{p2}$ would increase as $d_s$ increases. This boundary value is numerically identical to that in the original case, however, the gradients of the derivatives differ between the two cases as seen in Figure 23. The value of $\frac{\partial w_{p2}}{\partial d_s}$ increases much faster in the case with the subsidy-sharing policy implemented than when it is not.

Therefore, by implementing this subsidy-sharing policy on ports, the government makes it easier for port 1 to lower its service charges while reducing the amount that port 2's service charges increase, improving the overall financial performance of both shipping lines.



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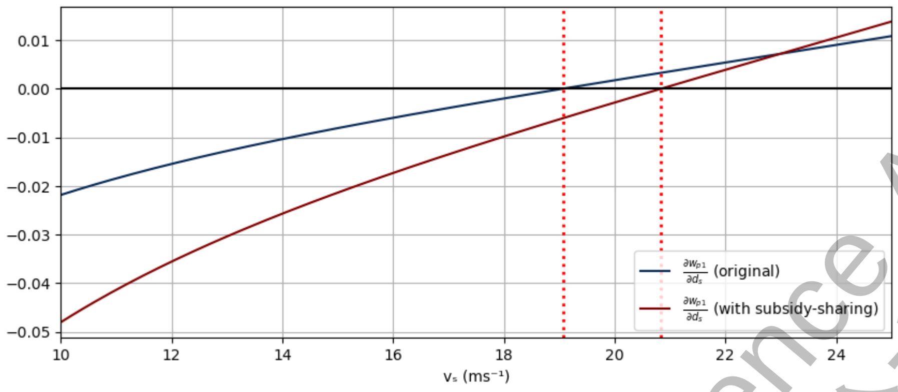


Figure 22: The relationship between $\frac{\partial w_{p1}}{\partial d_s}$ and $v_s$ with and without subsidy-sharing policy

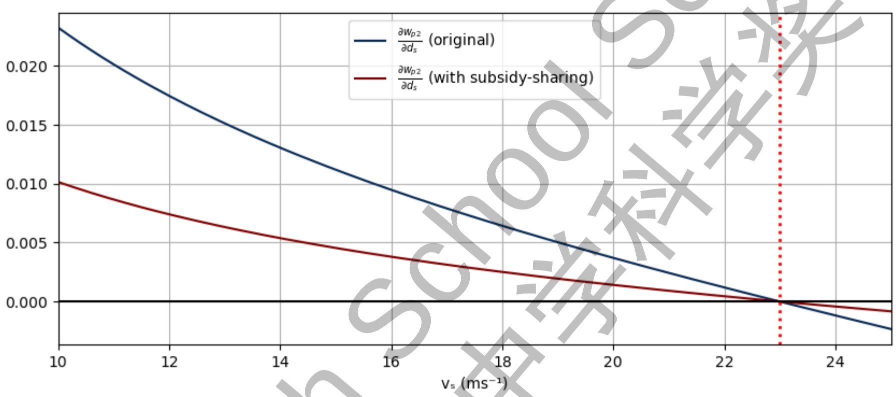


Figure 23: The relationship between $\frac{\partial w_{p2}}{\partial d_s}$ and $v_s$ with and without subsidy-sharing policy

## 5 Conclusion

This paper conducts an in-depth study of the bi-level game consisting of ports and shipping companies. First, we utilized an evolutionary model to examine the behaviors of ports and shipping companies under a vessel speed reduction subsidy (VSRS) policy over time and how different operating variables affect the final strategy each party adopts. Next, a two-channel game is constructed with only one channel having implemented a VSRS policy where ports offer subsidies to vessels navigating at reduced speeds within the speed reduction zone (SRZ). Modeling the relationship between the shipping companies with a Nash game and a Stackelberg game, we analyze how VSRS impacts the operations of the ports and shipping companies, as well as the ports' emissions, under both scenarios. Finally, we extend the model under the Nash game scenario to become a tripartite

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game involving also the government. Specifically, we investigate the effects of a subsidy-sharing government policy and explore its effect on the shipping line.

The main findings in this paper are as follows:

(1) From the results of the evolutionary game, we find that the factors influencing the shipping company's evolutionarily stable strategy include but are not limited to: the subsidy amount provided by the port, the extra profits gained from re-routed ships docking at the port, the opportunity costs of slowing down due to time costs, and the opportunity costs of not slowing down due to the subsidy forgone. The price of fuel does not have a notable influence on the choice of strategy, only affecting the speed at which the strategy converges. On the other hand, the port's evolutionarily stable strategy is only affected by mainly 1 variable which is the extra profits from re-routed ships, and its strategy's speed of convergence slightly by the subsidy amount. Specifically, when the extra profits from re-routed ships are at certain values in the middle, it produces a periodic oscillation in both the port's and shipping company's strategy, where their strategies constantly switch from one to another, possibly engendering large uncertainties in the shipping lines.

(2) From the dual-channel game in Section 3, we firstly find that as long as the speed limit is set below the original optimal vessel speed for the shipping companies, the freight rates of the shipping line that implements the VSRS decreases as the size of the SRZ increases. The freight rates of the other shipping line, however, increase as the size of the SRZ increases. An increase in the speed limit will also help lower the freight rates of the shipping company whose corresponding port implements VSRS and raise that of the other. For the ports, the SRZ radius and speed limit influence the port service charges similarly as it does to freight rates but with differing boundary values. Overall, the port service charges are less easily decreased by increasing the speed limit than the freight rates. Moreover, the differing boundary speed limit values for the two ports' service charges cause there to exist a set of values for the speed limit that when the size of the SRZ increases, both ports' service charges would increase, hence harming both shipping lines financially. For shipping companies' profits, the implementation of VSRS has the opposite effect on them compared to the freight rates and port service charges. Compared to the N-N scenario, the freight rates, port service charges, and shipping company profits are all much higher in the N-S scenario. Finally, for emissions, increasing the speed limit at a port implementing the VSRS causes an increase in emissions for itself while decreasing emissions for the competing port. Notably, the effects of VSRS on emissions are more

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prominently demonstrated in the N-S scenarios where the port implementing VSRS experiences a larger decrease in emissions compared to the N-N scenario.

(3) When the government intervenes, under a subsidy-sharing policy, the positive effects on the shipping line that implements VSRS are accentuated whilst the negative effects on the competing shipping line subsides. Overall, the subsidy-sharing policy helps a VSRS policy further elevate the financial performance of both shipping lines and also increases the incentive for the port to increase its SRZ radius, further lowering the emissions. Whilst policies as such can benefit the shipping lines, they also pose risks of causing severe disruptions to the market dynamics, possibly worsening the already large uncertainties in the maritime industry.

All in all, this study yields significant results in understanding how emission regulations implemented in maritime supply chains can affect their operations and emissions. However, several areas could still be improved in future works, including (1) Investigating how uncertainties in fuel prices affect the operations of these parties within a maritime supply chain. This paper assumes the value to be constant, however, fuel prices in reality are highly volatile and, hence are likely able to affect the decisions made by these shipping lines. (2) Modeling the effects with non-linear demand. For simplicity, this study assumes the demand function to be linear. However, in reality, the demand is rather non-linear, especially due to the various disruptions faced by this industry and inherent characteristics such as the bullwhip effect. (3) Performing case studies on specific ports. This paper mainly provides the theoretical framework to study vessel speed reduction policies, but it does not perform targeted case studies. Future studies can perform similar analyses on specific ports and thus give tailored policy suggestions and guidance for their port of choice. Additionally, for such models to get closer to seeing implementation in the real world, even more realistic modeling techniques and assumptions would have to be made. Lastly, we hope this area will gain the higher recognition levels across the globe it deserves and further studies will be conducted to improve this urgent climate crisis at hand.

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# Appendix

Section 2

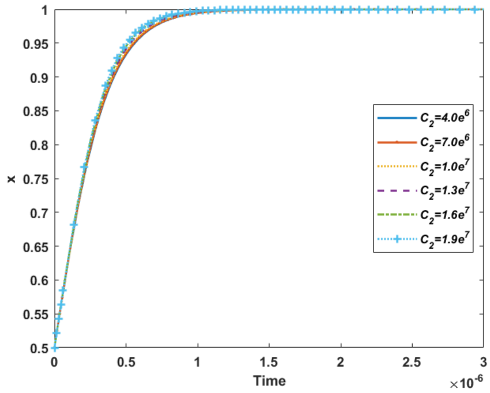


Figure 24: Effect of different $C_2$ values on the evolution of $x$

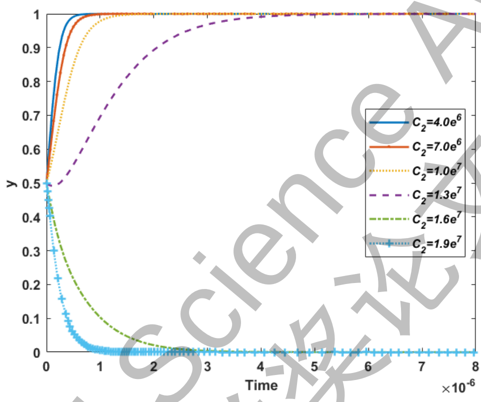


Figure 25: Effect of different $C_2$ values on the evolution of $y$

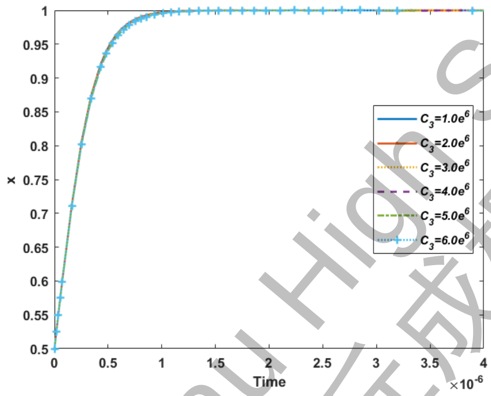


Figure 26: Effect of different $C_3$ values on the evolution of $x$

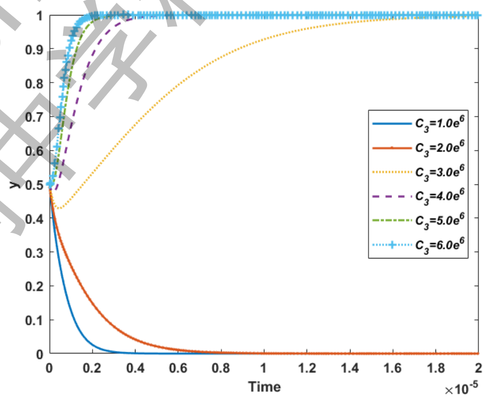


Figure 27: Effect of different $C_3$ values on the evolution of $y$

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## Acknowledgement


### Topic Origin

With my father being an experienced trader and my sister studying finance and management for her master's degree, I grew up in an environment where most of the conversations with my family were linked to finance in some way or another. From the early age of 8, I was introduced to trading stocks. I would often go with my father to work and even attend company meetings on market and stock analysis. Over time, I have caught on to his habit of over-exploiting technical analysis and neglecting the more macro side of things. Being an amateur trader/investor myself, realizing the importance of macroeconomic aspects in investing and trading, I now often look into news and reports about the wider economy to maintain a secure and safe footing to base my trades upon. This is where I came across the global issue of supply chain breakages. Especially with news on various maritime supply chain issues such as the congestion at the Port of Shanghai during COVID-19 or the blockage of the Suez and Panama Canals, I have always kept a lookout for this area. Through further research, I came across the various emission regulations and policies faced by these shipping lines. Hence, watching this industry being placed between a rock and a hard place, I decided to set my research at the junction of these two concerns, considering the complex interactions between emission regulations and maritime supply chains' operations.

### Research Process

I started this research by extensively reading literature on supply chain management without a specific focus on the maritime industry. It was thanks to Mr. Wu's constant guidance and the questions he often asked me about my knowledge in this area that our research topic was then fully sculpted. With my eagerly profound interest in game theory, I quickly saw the harmonic applicability of such methods in maritime supply chains. Due to the time-bounded nature of many emission goals and green initiatives, I realized evolutionary game theory may be a good fit for our problem, hence taking up the first section of our study. During the evolutionary result analysis part, even with my background in Python, performing this specific type of data analysis posed a challenge. Mr. Wu then prompted me to learn MATLAB which sure simplified the process a lot as well as enriching my skillset in data visualization. In the second section of the study, considering the financial aspects


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of maritime supply chains, I proposed to find the optimal operational parameters of different players and investigate the effects of emission regulations on them. To further add to the financial aspect of this, I decided to compare the cases of different competition structures as relevant literature often made strong assumptions about such characteristics. Seeing my paper being so theoretical, Mr. Wu reminded me to not neglect the importance of practicality in research, leading to the addition of the more grounded numerical analysis and government sections.

Ultimately, I would like to again express gratitude toward my instructor Mr. Wu for his altruistic help, guidance, and genuine care for my work in this entire research process. I am also hugely grateful to Tsinghua University and Tsinghua University Yau Mathematical Science Center for providing such opportunities to foster interests and development in high school students. Lastly, I would also like to thank my parents and family members for such a nurturing environment and for supporting me along the way. I would like to express gratitude to my instructor Mr. Wu for his altruistic help, guidance, and genuine care for my work in this research. I am also hugely grateful to Tsinghua University and Tsinghua University Yau Mathematical Science Center for providing such opportunities to foster interests and development in high school students. Lastly, I would also like to thank my parents and family members for such a nurturing environment and for supporting me along the way.

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