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Research Report

The Team

Registration Number: **Phy-145**

Name of team member: **Lin, yong- zhang**
School: **Kaohsiung Municipal Kaohsiung Senior Highschool**

Name of team member: **LIN, Jun-Yi**
School: Taipei Municipal Chien Kuo High School

Name of team member: **Lo, ta- chao**
School: **Kaohsiung Municipal Kaohsiung Senior Highschool**

Name of supervising teacher: **Lu, Cheng-Liang**
Job Title: **Physics Teacher**
School: **Kaohsiung Municipal Kaohsiung Senior Highschool**

Title of Research Report

Wetting tracing paper curling behavior and penetration of fiber porous media

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We hereby declare that we

1. are fully committed to the principle of honesty, integrity and fair play throughout the competition.
2. actually perform the research work ourselves and thus truly understand the content of the work.
3. observe the common standard of academic integrity adopted by most journals and degree theses.
4. have declared all the assistance and contribution we have received from any personnel, agency, institution, etc. for the research work.
5. undertake to avoid getting in touch with assessment panel members in a way that may lead to direct or indirect conflict of interest.
6. undertake to avoid any interaction with assessment panel members that would undermine the neutrality of the panel member and fairness of the assessment process.
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8. observe all rules and regulations of the competition.
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(Signatures of full team below)

X 林永章
Name of team member:

X [illegible]
Name of team member:

X 羅大釗
Name of team member:

X
Name of supervising teacher:
盧啟良

Noted and endorsed by

(signature) 莊福泰

Name of school principal:

Wetting Tracing Paper Curling Behavior and Penetration of Fiber Porous Media

Lin, Yong-Zhang LIN, Jun-Yi Lo, Ta-Chao

Contents

Abstract

This study delves into the curling behavior of tracing paper when exposed to water, a phenomenon of interest due to its relevance in various applications involving fibrous materials. Tracing paper, treated with sulfuric acid for translucency, exhibits a distinct response to moisture characterized by an immediate curling followed by a gradual uncurling. Our research combines experimental techniques and theoretical models to investigate these processes. We utilized slow-motion videography to capture the paper's deformation and employed Gaussian curvature measurements alongside Richards' equation to model the dynamics of water absorption and evaporation.

Key findings reveal that the paper's rapid curling is driven by differential swelling across its surface, resulting in a pronounced cylindrical shape. This initial deformation is influenced by the paper's unique pore structure and the directional nature of its cellulose fibers. The uncurling phase, characterized by a slower, temperature-dependent relaxation, aligns with exponential decay, suggesting that evaporation and water diffusion within the paper are crucial factors. Notably, increasing the temperature accelerates both the curling and uncurling processes, enhancing peak curvature due to elevated diffusivity and evaporation rates.

Furthermore, the study explores the impact of saline solutions, discovering that higher salt concentrations lead to more pronounced curling. This is attributed to the salt's influence on the paper's mechanical properties and the rate of water absorption. These insights contribute to a nuanced understanding of how fibrous materials interact with water, particularly under varying environmental conditions.

Keywords: Capillary, Richard's Equation, Diffusion, Porous Media, Fiber,

1. Introduction

When a tracing paper is gently placed on a water surface, the paper rapidly curls and then slowly uncurls. Investigating this phenomenon is significant as it provides insights into the interactions between water and fibrous materials, specifically cellulose fibers. Tracing paper, which is often treated with sulfuric acid, has a dense, low-connectivity pore structure. This structure influences how water penetrates and spreads within the material, showcasing the paper's physical and chemical properties during the wetting process. Additionally, tracing paper is sensitive to humidity, which can further affect its interaction with water. Understanding this behavior has practical implications, especially in evaluating how industrial fibrous papers with curl issues are affected by water interaction

1.1 Observations:

Experimental Setup:

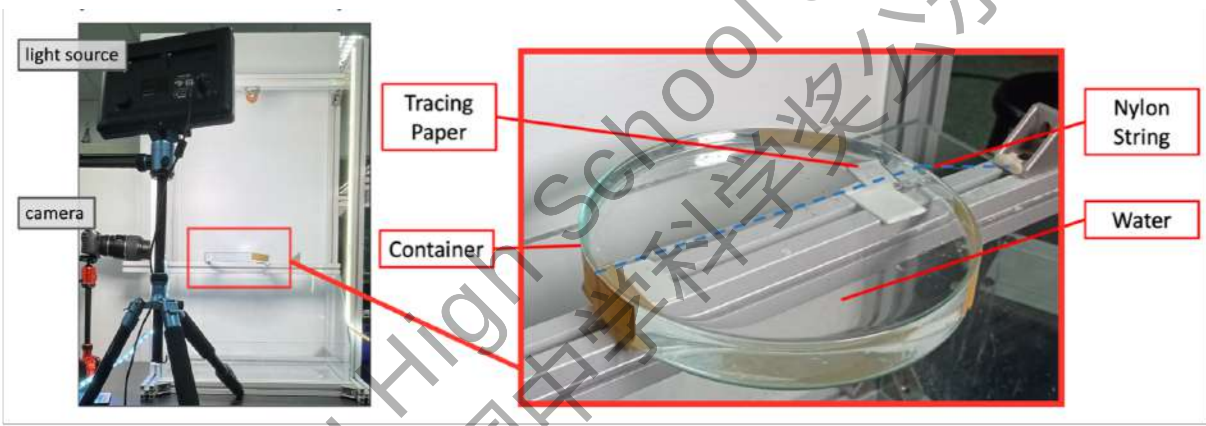


Figure 1: Here we use a container to hold the water. We suspend a nylon string directly on the surface of water to pin the tracing paper in place. When putting tracing paper, we lift the nylon string above the water and use tweezers to place the tracing paper under the string.

Preliminary Observation:

When the tracing paper was placed on the water's surface, it was observed that the paper initially curled into a cylindrical shape before gradually and slowly uncurling. To better understand this phenomenon, the curvature of the tracing paper was analyzed over time.

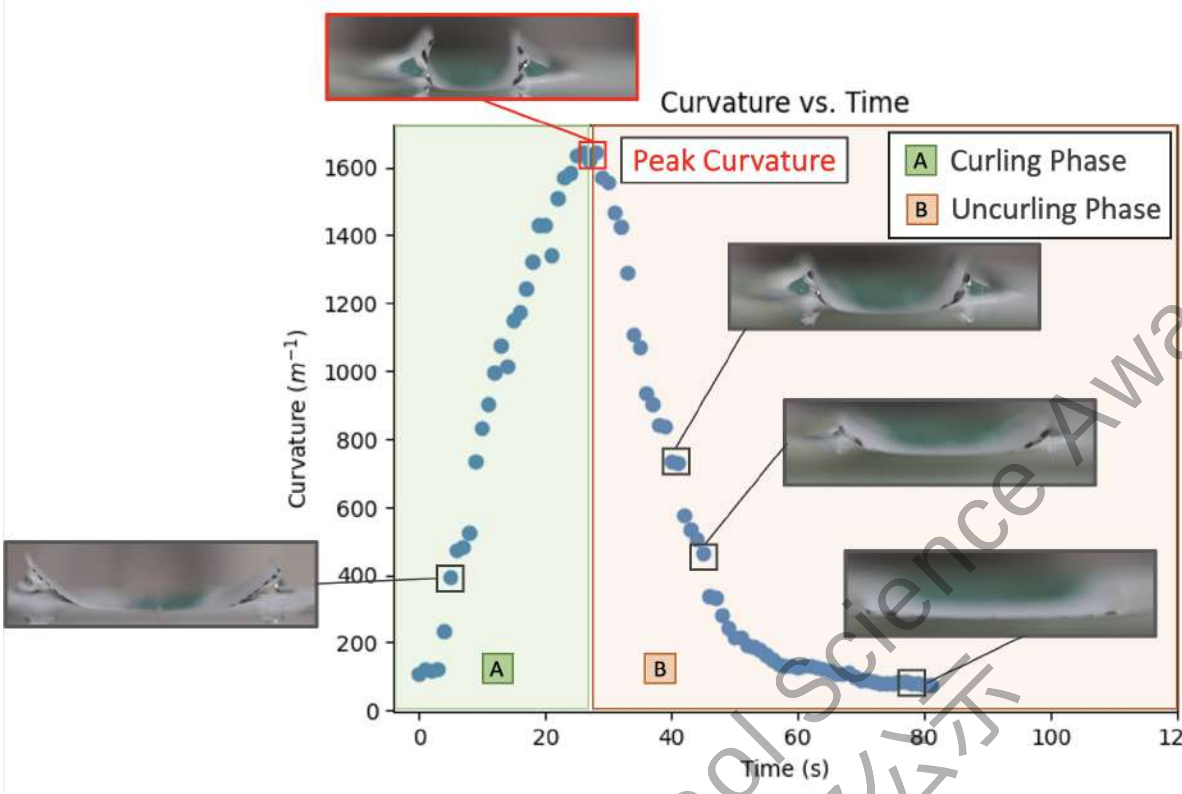


Figure 2: From the result we observed that

Analysis-Curvature

The curvature of tracing paper interacting with water was analyzed using a slow-motion camera positioned parallel to the cylindrical shapes formed during curling. Gaussian curvature ($\mathcal{K} = \kappa_1 \cdot \kappa_2$) was used to describe the surface's curvature, where κ_1 and κ_2 are the principal curvatures. As the paper curled into a cylindrical shape, it exhibited Zero Gauss Curvature because one curvature was non-zero ($\kappa_1 \neq 0$), while the perpendicular curvature was zero ($\kappa_2 = 0$). This results in a developable surface, characteristic of cylindrical shapes.

1.2 Pore and Fiber Aspect-Droplet Experiment

Since tracing paper is a fibrous and porous material(celluouse fiber, we concern that the mechanism of capillary of capillary imbibition in two aspects:

To clarify that the capillary mechanism is in the result of whether pore or fiber concerns, we initially did an experiment(Droplet Experiment). Here, we use a pipetmen in 10μm amount of water(We chose such a small amount of water specifically to minimize the influence of gravity) then drop on tracing paper, forming a droplet. Then use a digital microscope to observe the droplet on hydrophobic and hydrophilic concerns.

When a water droplet comes into contact with tracing paper, the contact angle provides critical insight into the interaction between the water and the paper's fiber network. If the fibers are hydrophobic, the capillary action within the fiber web of the tracing paper is significantly reduced. This occurs because hydrophobic fibers repel water, preventing the liquid from infiltrating the microstructures of the paper. As a result, the typical capillary forces that would otherwise draw water into the fiber network are absent, leading to minimal or no swelling and a lack of observable wrinkling. Conversely, if the fibers are hydrophilic, the capillary action is more pronounced, allowing water to be absorbed into the fiber network. This absorption induces swelling as the water infiltrates and expands the cellulose fibers, leading to noticeable wrinkling and deformation of the paper.

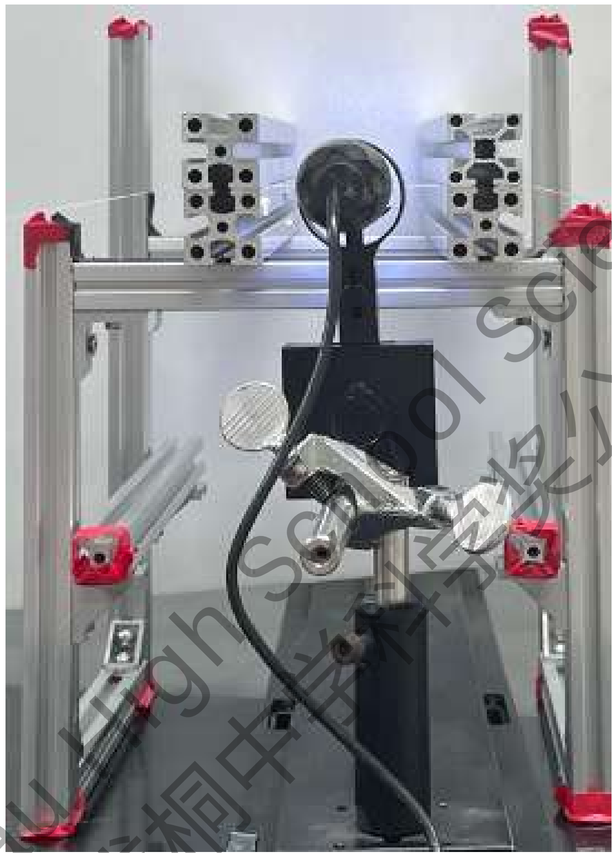


Figure 3: Expermential Setup

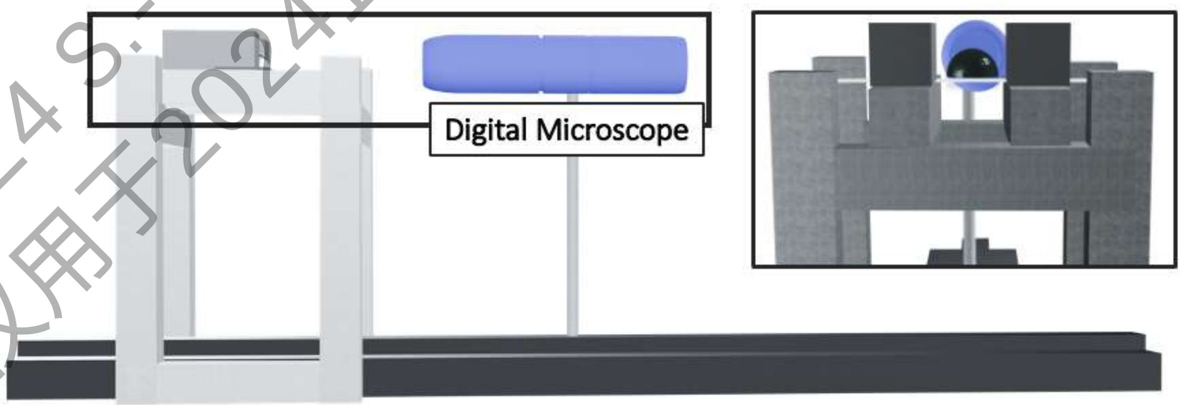


Figure 4

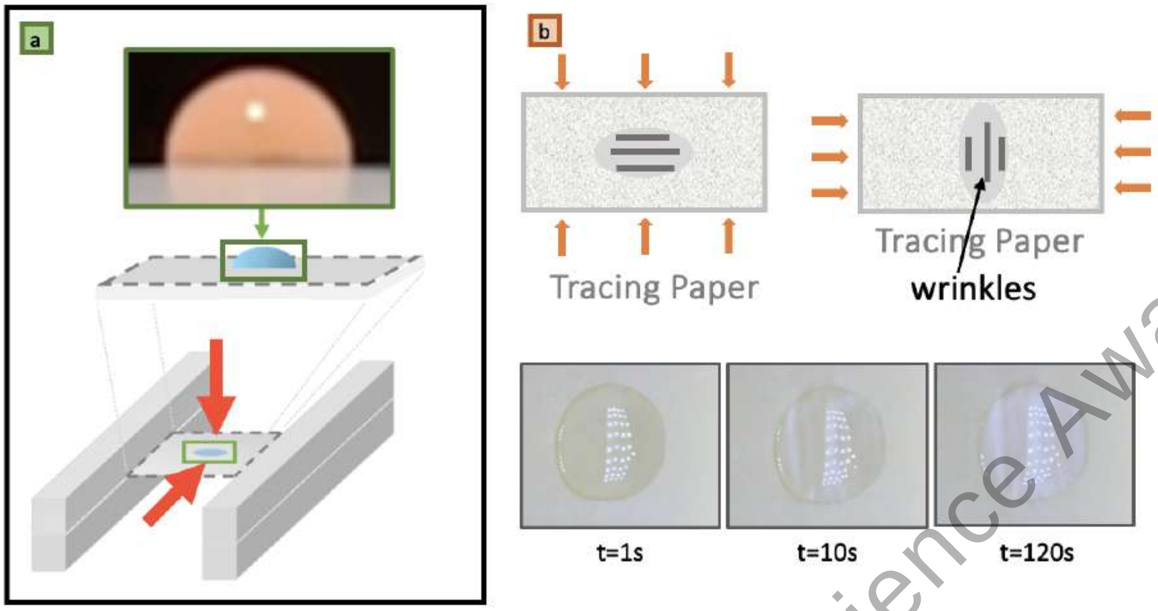


Figure 5: From fig(a) we can see that the 10 um droplet under digital microscope is hydrophobic. However, we still can see tracing paper swell after a period of time, in fig(b) below is a 10 ml of droplet on tracing paper after time, which can better show the description that mentioned. Also, we found out the curling direction of tracing paper when placed on water surface is always perpendicular to the direction of the wrinkle, which recalling to that curling direction is CD Direction that **1.2.5** discussed further.

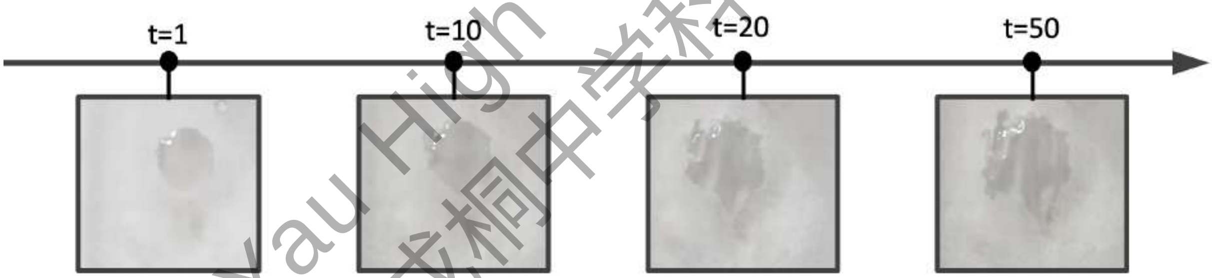


Figure 6: Here we use sandpaper to rub tracing paper then again drop a water droplet diffuse after time

According to out experimental result, we qualitatively says that tracing paper is hydrophobic on the outer layer and hydrophilic in the inner layer.

Swelling Mechanism

1.2.1 Introduction

The swelling mechanism of tracing paper is a complex interplay between its hydrophobic outer layer and hydrophilic inner layers. Initial water resistance is overcome through capillary action, allowing water to reach the absorbent cellulose fibers beneath the surface. This process is primarily driven by pore sorption, suggesting that while the outer layer repels water, the inner structure's hydrophilicity leads to eventual swelling. Understanding this mechanism, particularly the role of coatings and the dual-layer structure, is essential for applications where the moisture resistance and swelling behavior of tracing paper are critical factors.

1.2.2 Observations and Hydrophobic Nature of Tracing Paper

In recent droplet experiments conducted on tracing paper, a unique swelling mechanism was observed that did not align with typical water absorption behaviors seen in other cellulose-based papers. Both vertical and perpendicular applications of water droplets revealed that the water front did not diffuse after a prolonged period, indicating a distinct interaction between the tracing paper's surface and water. This behavior highlights the hydrophobic characteristics of the outer layer of tracing paper, which plays a crucial role in its initial resistance to swelling.

The hydrophobicity of tracing paper is primarily due to its outer layer, which has been treated or coated to repel water. Hydrophobic surfaces typically resist water penetration, leading to delayed or minimal swelling. This behavior can be explained through the mechanism of hydrogen bonding, which governs the interaction between water and cellulose fibers. According to Klemm et al. (1998), the hydrogen bonds within cellulose fibers are crucial in determining water absorption properties. In cellulose, the hydroxyl groups form hydrogen bonds with water molecules, facilitating water absorption and subsequent swelling. However, when a hydrophobic coating is applied to the surface, these interactions are significantly hindered, preventing immediate swelling upon contact with water.

1.2.3 Inner Hydrophilic Layer and Capillary Action

Despite the initial resistance due to hydrophobicity, tracing paper does eventually swell over time, indicating that its inner layers are hydrophilic. This delayed swelling suggests that water penetration is not primarily due to the fibers themselves but rather through capillary action within the pores of the paper.

This capillary action, driven by the pore sorption mechanism described by the Richardson equation, allows water to be drawn into the microscopic pores of the paper, bypassing the hydrophobic barrier. Initially, it was assumed that the swelling mechanism was dominated by fiber sorption, where water enters the fiber through parallel cylindrical capillaries, leading to swelling. However, since tracing paper exhibits hydrophobic properties, this fiber sorption mechanism appears insufficient. The hydrophobic nature suggests that while the outer layer of the paper repels water, the inner layers may possess hydrophilic characteristics. As water penetrates through capillary action into these inner layers, it causes the cellulose fibers to absorb water, resulting in swelling.

1.2.4 Coating and Hydrophobicity Considerations

The hydrophobic nature of the outer layer in tracing paper is often a result of specific treatments or coatings applied during its manufacturing. According to Kjellgren (2007), the barrier properties of greaseproof paper, including tracing paper, are significantly influenced by polymer coatings applied to enhance their resistance to moisture and grease. These coatings contribute to the paper's hydrophobic characteristics, preventing water from penetrating the surface and thus delaying swelling. However, once the water overcomes this barrier, likely through capillary action into the pores, it reaches the inner, hydrophilic layers, where it can cause swelling.

1.2.5 Curling Direction-Fiber Orientation

In order to use a string to put the paper in place, we shall first predict in what axis the paper will curl; otherwise, the string may affect its curling motion.We define the curling direction as the axis of the cylinder formed by the tracing paper. It is seen by cutting the piece of paper at different angles that the axis of scrolling is always parallel to one of the original edges. (observation, Side A,B C will describe in Figure Two)

By observing the expansion of a piece of fully wet paper, we see that it displays an apparent anisotropy in two perpendicular directions parallel to its original edges.

The direction with higher expansion corresponds to the Cross Direction (CD), which is perpendicular to the fiber orientation direction, Machine Direction (MD) **DeRuvo1973**.)

These terms originate from the manufacture of paper. As intuition would suggest, the height of the cylindrical shape produced by scrolling is perpendicular to CD, indicating that the direction with higher expansion dominates the scrolling.

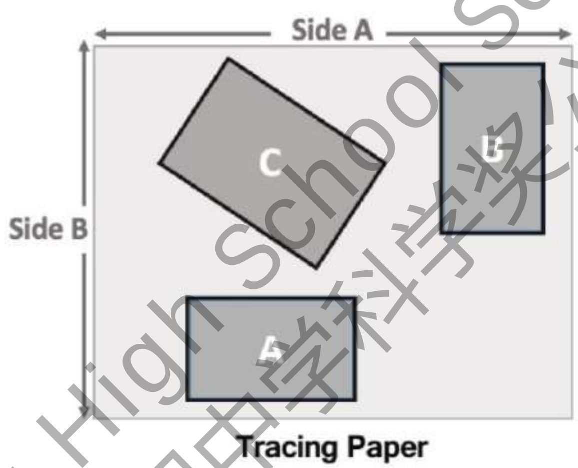
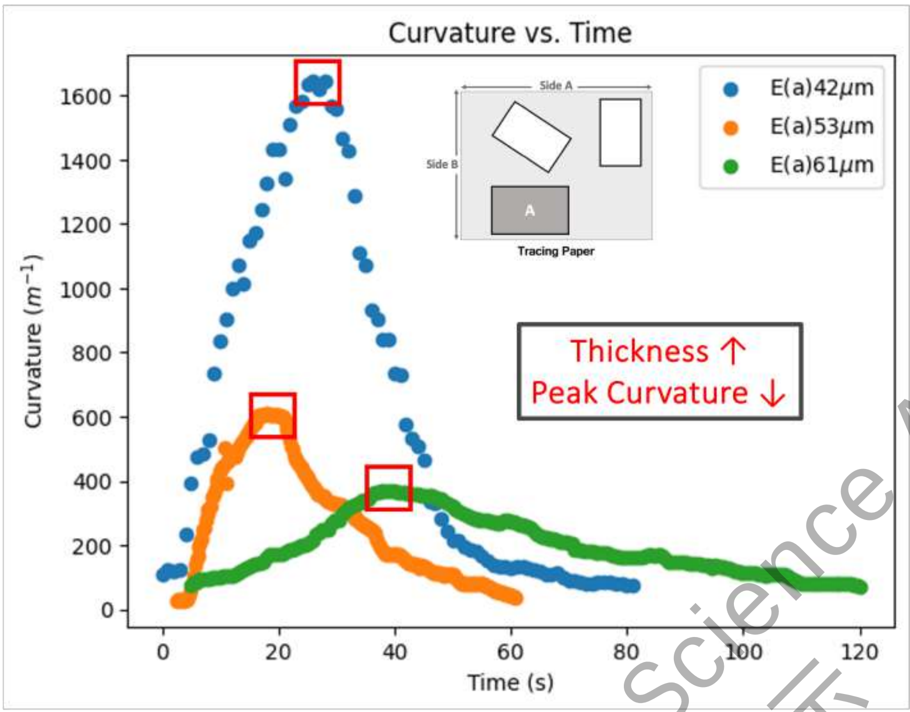
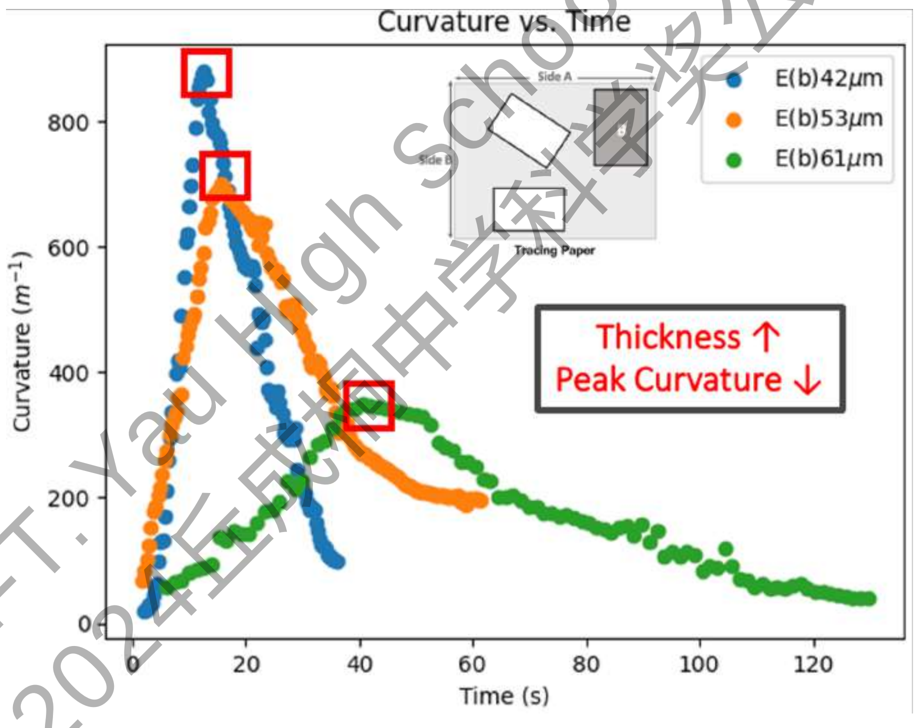


Figure 7: Cut an unaltered piece of paper direct from manufacture in different angles. Their axis of curling are all the same, parallel to the Side B. In this case, Side A corresponds to CD, Side B is MD.


Curvature vs. Time
Curvature (m^{-1})
Time (s)
E(a)42μm
E(a)53μm
E(a)61μm
Side A
Side B
A
Tracing Paper
Thickness ↑
Peak Curvature ↓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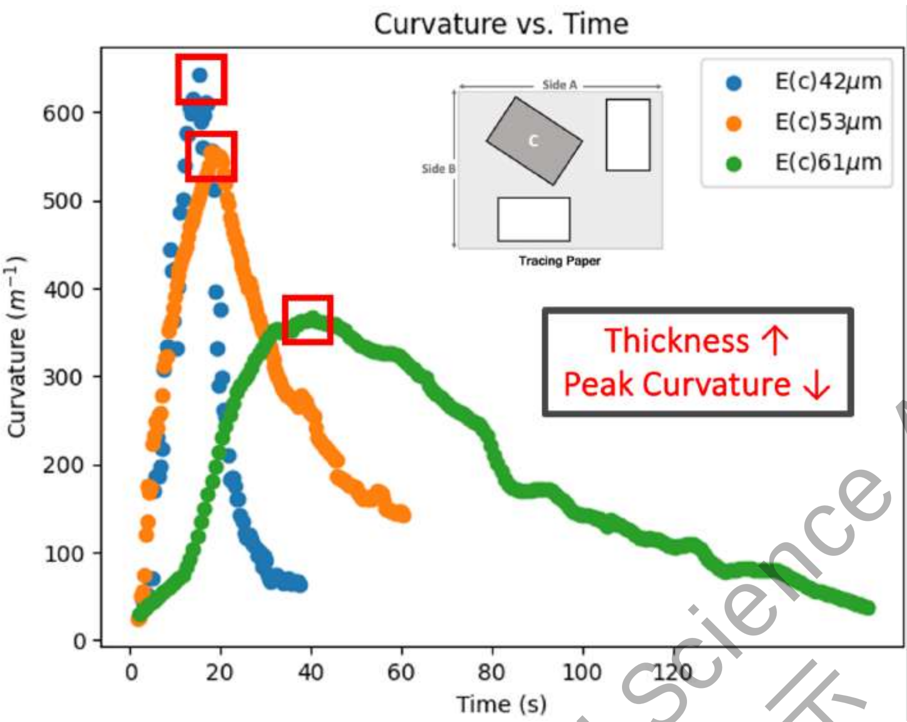


Figure 8: The three graphs are that we tracking curvature of tracing paper in A,B,C directions under different thickness which in Fig.6 mentioned. From the experimental result of curvature to time we can found out two key insights. Firstly, from the three graph we all can found out that when thickness increases peak curvature decreases, peak time increases, which will be qualitatively explained in **parameters discussion** further. Secondly, from the three graphs we can also find that curvature with same thickness in different angle(A,B,C) is different, since tracing paper have been cut in different angle, the MD/CD length differs.

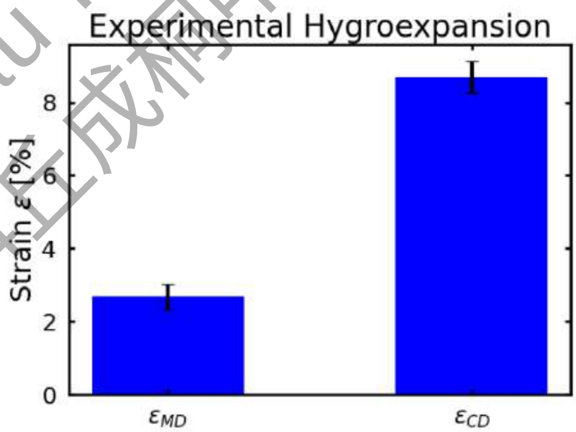


Figure 9: The saturated expansion rate (strain) of a 10x10mm piece of paper is found. It differs greatly in two perpendicular directions.

1.2.6 Cylinders Formation

Depending on different initial conditions, the geometry the tracing paper will produce vary. If the CD length is less than the circumference produced by the peak curvature reached, then the two ends of tracing paper will not touch, which we call the 0-scroll. This case is the main phenomenon of study, since it is minimally affected by factors such as friction. If the CD length is bigger than the threshold, then it will curl into a cylindrical shape, dubbed 1-scroll. Whether it sinks at the peak curvature or not depends on whether the gravity exceeds surface surface tension, and they are in turn related to the ratio of MD and CD length and the peak curvature. If the paper is not released perfectly flat, then the two ends will curl at a different time and amount, thus one end will curl and touch the water, producing a scroll in opposite direction, and finally sink and uncurl underwater. If the paper is released flat enough such that the two ends collide and force each other into cylinders, it will form two cylinders, which we call the 2-scroll.

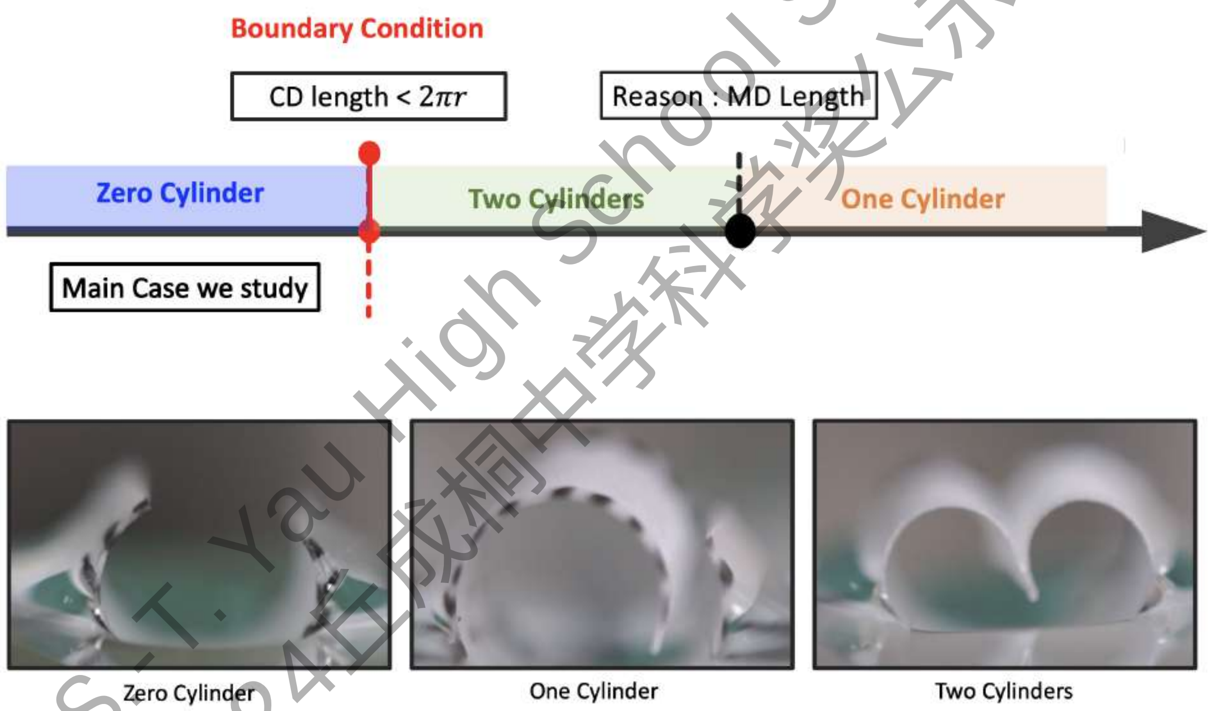


Figure 10: Cylinders Formation

Figure 11: Failure Case

2. Quantitative

2.0.1 Diffusion Model Discussion

It is argued that capillary effect is not involved by showing that surfactants does not affect the curvature (Reyssat and Mahadevan 2011). However, it should be noted that surfactant molecules cannot penetrate cellulose surfaces in aqueous solutions (Penfold et al. 2007). Since surfactant molecules cannot affect the water that goes into the paper, the capillary effects must not be ruled out. Moreover, we could see later (9) that the mathematical representation of Fick's law is merely a limiting case of the Richards equation. This suggests a broader view be obtained when capillary effect is considered, rather than assuming a limiting case.

2.0.2 Capillary Model Discussion

The capillary model proposed by (Washburn 1921) is well-known and is applicable to many cases of porous materials. However, (Reyssat and Mahadevan 2011) proposed that this model failed to account for the whole phenomenon. Literature (Huinink, Ruijten, and Arends 2016; Perez-Cruz, Stiharu, and Dominguez-Gonzalez 2017) shows that a key assumption of the Washburn law is the binary distribution of water content in a porous media, and we think this is why the model fails to explain the results. To improve on this, (Huinink, Ruijten, and Arends 2016) proposed a model that accounts for the gradual transition of water and air regime, allowing non-sharp boundary of water and air.

To begin the discussion of capillary model, describing how water moves in paper is critical. To quantitatively describe how much water is in the paper at a specific location z and time t, we follow (Huinink, Ruijten, and Arends 2016)volumetric water content as the ratio of water and the total volume,
We define the volumetric water content as the ratio of water and the total volume **thebibliography**.

$$\theta = \frac{V_{H_2O}}{V_{H_2O} + V_{cellulose}} \tag{1}$$

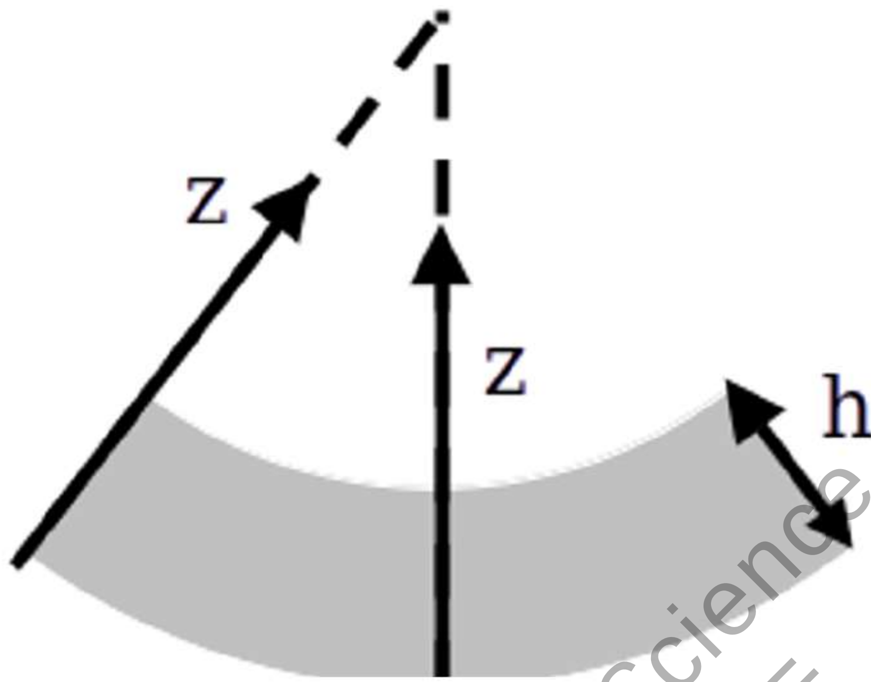


Figure 12: The coordinate axis of the paper. The origin of z axis is set at the water surface, and the direction is perpendicular to local tangent plane of paper.

Here, Fig.12 shows that the water content is dependent of z, but not in the x and y axis of paper because of symmetry.

Water content is seen to linearly increase the strain of paper without producing recovering stress. According to **Nissan1976**, Young's modulus decreases exponentially with volumetric water content. The decay coefficient may be different for various materials, and it is fitted in our case to be -24.32.

$$E(\theta) = E_0 e^{-24.32\theta} \tag{2}$$

By capillary reasoning, Richards equation can be used to model water penetration through porous media **Huinink2016**.

$$\frac{d\theta}{dt} = D_0 \frac{d}{dz}\left(\theta^n \frac{d\theta}{dz}\right) \tag{3}$$

in which D_0 is called "diffusivity" for convenience. It should be noted that D_0 is of capillary nature, and there is no diffusion involved. The exponent n takes a value of,

$$n = 1 + \frac{2\lambda}{\lambda} \tag{4}$$

Where λ is the pore distribution index **Huinink2016**, which is smaller when the pore sizes are near, we arrive that,

$$D = D_0\theta^n, \quad n = 2 + \frac{1}{\lambda} \tag{5}$$

D_0 can also be written explicitly,when cylendrical pores is assumed

$$D_0 = \frac{\gamma \cos \alpha r}{4\mu\lambda} \tag{6}$$

When trying to fit the experimental data, (Perez-Cruz, Stiharu, and Dominguez-Gonzalez 2017) uses an exponential value of 1.1, which is contradictory against what (Huinink, Ruijten, and Arends 2016) had proposed. We think it may illuminate the fact that the assumption of cylindrical pore does not apply.

2.1 Richard's Derivation

The path of **Huinink2016** would be followed to derive Richards equation. The law of mass conservation is well-known,

$$\frac{\partial \theta}{\partial t} = -\frac{\partial q}{\partial x} \tag{7}$$

Since paper is a porous media, Darcy's law applies,

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(\frac{k(\theta)}{\mu}\left(\frac{\partial p(\theta)}{\partial x} + \rho g \hat{z}\right)\right) \tag{8}$$

Where k is the permeability involved in Darcy's law, μ is the liquid viscosity, p is the pressure in the pore, ρ is the liquid density, g is the gravitational acceleration, $\hat{z}$ is the unit vector in the direction of gravity. It is known that the pressure can be written as

$$p = p_0 - p_c(\theta) \tag{9}$$

Where p_0 is the equilibrium pressure of liquid and air, p_c is the pressure provided by capillary action. Rewriting the equation and ignoring gravity,

$$\frac{\partial \theta}{\partial t} = -\frac{\partial}{\partial x}\left(\frac{k(\theta)}{\mu}\left(\frac{\partial p_c(\theta)}{\partial x}\right)\right) \tag{10}$$

Packing the coefficients,

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(D\frac{\partial \theta}{\partial z}\right) \tag{11}$$

With the value of D given by,

$$D = -\frac{k}{\mu}\frac{\partial p_c}{\partial \theta} \tag{12}$$

Given the simplified Brooks-Corey relationship,

$$p_c(\theta) = p_{c0}\theta^{-\frac{1}{\lambda}}, \quad k(\theta) = k_{\max}\theta^{3+\frac{2}{\lambda}} \tag{13}$$

2.2 Mere Limiting Case-Diffusive Mathematical Formula

It is seen now that the diffusive mathematical formula(Fick's law) can be seen as a mere limiting case of the Richards equation (10) at late time, when the water content is close to saturation everywhere.

$$\frac{\partial \theta}{\partial t} = D_{\text{sat}} \frac{\partial^2 \theta}{\partial z^2}, \quad D_{\text{sat}} = D_0 \theta_{\text{sat}}^n \tag{14}$$

The boundary condition at the bottom (z=0) states that paper remain saturated in contact with water; at the top there may be effects of evaporation, which is not considered significant given the time duration of the problem. However, we consider it critical as it can account for the deviation for the uncurling phase.

$$\theta(0,t) = \theta_{sat}, \quad \frac{d\theta}{dz}(h,t) = \frac{q}{AD_0} \tag{15}$$

$\theta_{sat} = 0.33$ is the saturated volumetric water content, h is the paper thickness, q is the volumetric evaporation rate, A is the surface area of paper. θ_{sat} is measured by weighing the paper before and after dipping in water, and converted to volumetric ratio by density of water and paper, both measured in a short enough duration of time.

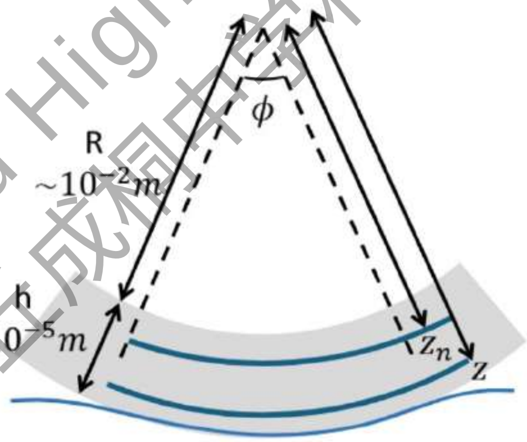


Figure 13: The geometry of curling paper.

For quantitative prediction of curvature, we consider the net strain (ϵ) and stress (σ) produced by curvature being canceled by the strain introduced by water penetration **Reyssat2011**.

$$\epsilon_\kappa = \kappa(z - z_n) \tag{16}$$

$$\epsilon_\theta = \epsilon_{sat} \frac{\theta}{\theta_{sat}} \tag{17}$$

$$\sigma = E(\epsilon_\kappa - \epsilon_\theta) \tag{18}$$

Where κ is the curvature, z is the position in the axis of paper thickness, z_n is the position of neutral layer, E is the Young's modulus. Assuming quasi-static process, we arrive at the prediction **Reyssat2011**.

$$\kappa = \frac{I_1 I_{\phi 0} - I_0 I_{\phi 1}}{I_1^2 - I_0 I_2} \tag{19}$$

$$I_0 = \int_0^h E\,dz, \quad I_1 = \int_0^h Ez\,dz, \quad I_2 = \int_0^h Ez^2\,dz \tag{20}$$

$$I_{\phi 0} = \int_0^h E\epsilon_\phi\,dz, \quad I_{\phi 1} = \int_0^h Ez\epsilon_\phi\,dz \tag{21}$$

Under this model, curvature is dictated by water content. Richards equation can only be used to predict normalized curvature **Perez-Cruz2017**. We follow the path and use only maximum curvature to adjust this. It is seen that Richards equation fits the curling phase really well, but cannot predict the uncurling phase (Fig. 2). Since at late time the diffusivity approaches a constant, we can use Fick's law to approximate late time behavior. We will use exponential function to approximate Fick's law **Reyssat2011**,

$$\kappa(t) \approx C\frac{\epsilon}{h}e^{-\frac{\pi^2 Dt}{4h^2}} \tag{22}$$

Where $C \approx 1.33$, D is the late time diffusivity. It is seen that it fit the data nicely.

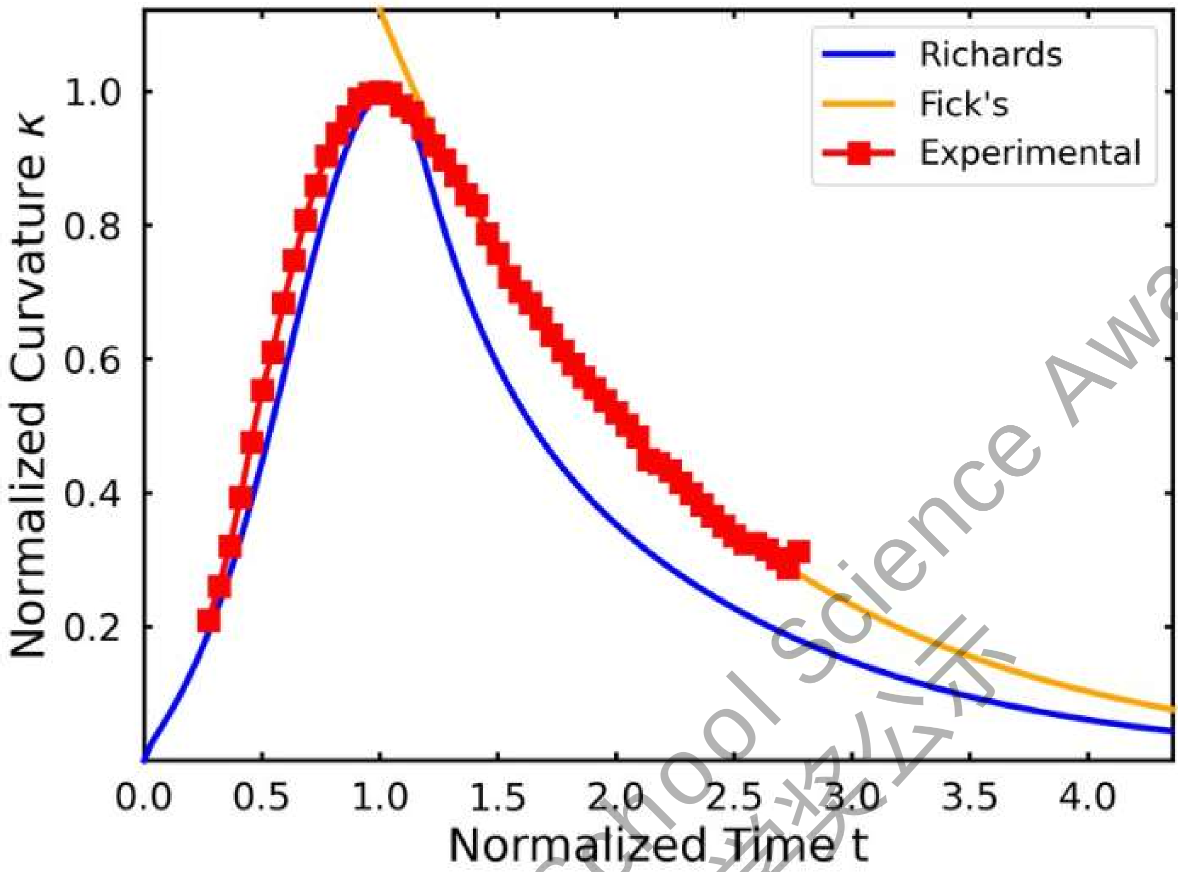
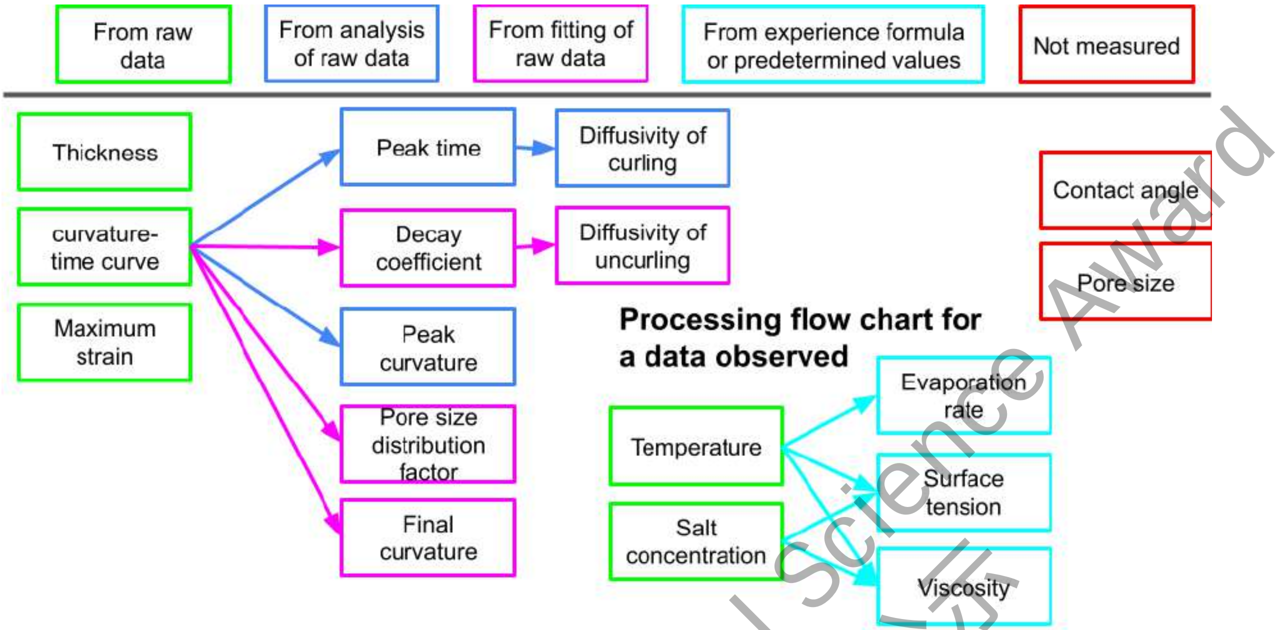


Figure 14: Enter Caption

2.3 Outline



2.4 Parameters Discussion

Paper dimension is fixed to be 10x10mm in length and width, and 87.15m in thickness if not specified. Paper thickness. By normalization of Richards equation, we see the time scale is (Perez-Cruz, Stiharu, and Dominguez-Gonzalez 2017),

$$\tau = \frac{h^2}{D_0 \phi_{sat}} \tag{23}$$

and the curvature scale is,

$$\kappa = \frac{\epsilon_{sat}}{h} \tag{24}$$

By simulation methods, we can deduce that the peak time and curvature is directly proportional to the scales,

$$t_{max} = 0.62\tau, \quad \kappa_{max} = 1.1\kappa \tag{25}$$

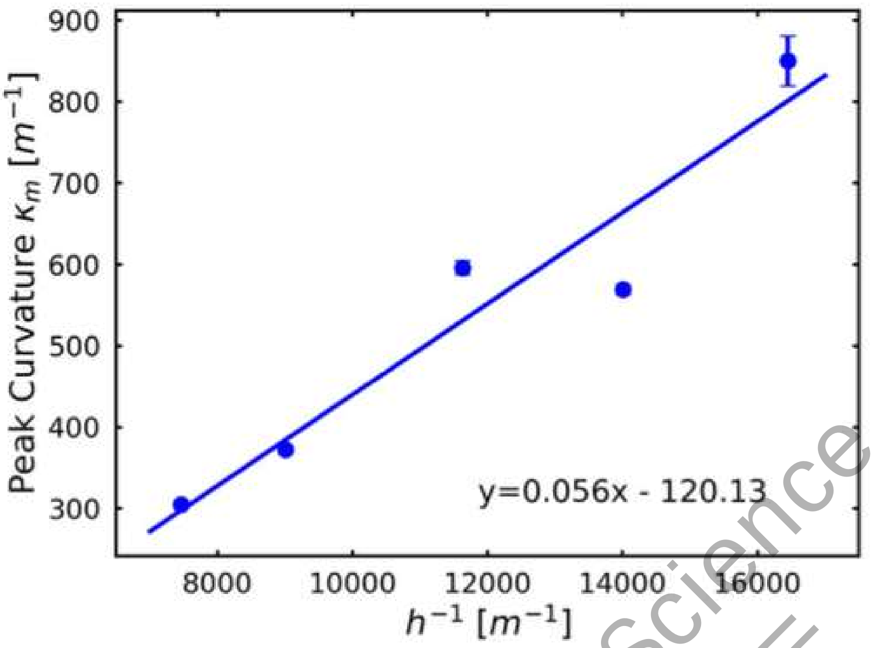


Figure 15: Linear relationship with peak curvature and reciprocal of thickness.

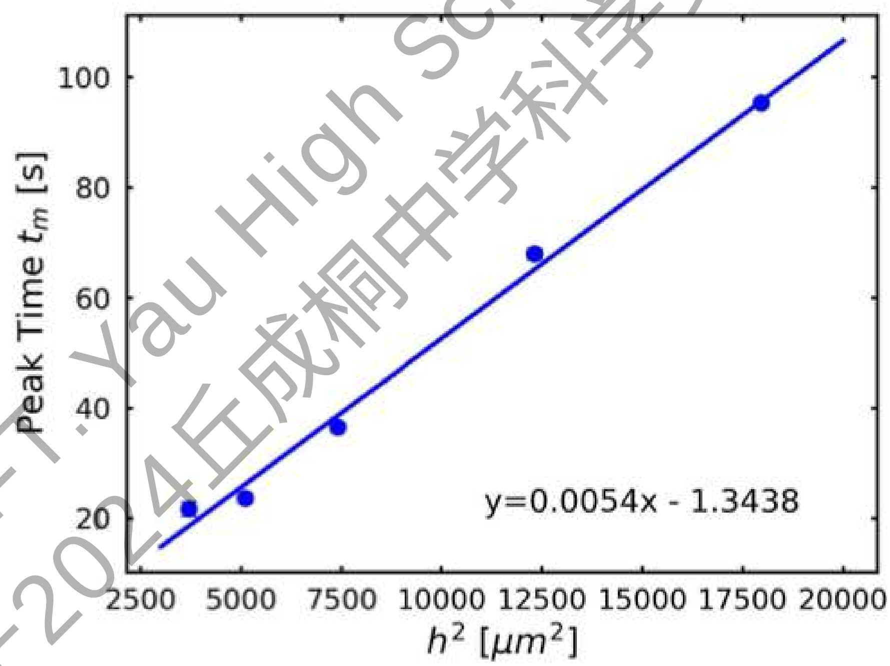


Figure 16: Linear relationship between peak time and thickness squared.

2.5 Temperature and Evaporation

Temperature itself affects a lot of factors, including Young's modulus, surface tension, viscosity, and evaporation rate. The absolute value of Young's modulus does not matter, since we assumed a quasi-static process. For surface tension and viscosity, it is mentioned that their ratio affects the diffusivity linearly. It is previously measured that temperature dependence of surface tension and viscosity of water can be described by the following formulae:

$$\gamma = a\left(\frac{T_c - T}{T_c}\right)^b \left(1 - c\left(\frac{T_c - T}{T_c}\right)\right), \tag{26}$$

$$a = 0.2358, \quad b = 1.256, \quad c = 0.625, \quad T_c = 647.15\,\mathrm{K}, \quad 273.01\,\mathrm{K} \le T \le 647\,\mathrm{K}$$

$$\mu = Ae^{B/T} + CT + DT^2, \tag{27}$$

$$A = 1.86 \times 10^{-14}, \quad B = 4209, \quad C = 0.04527, \quad D = -3.38 \times 10^{-5}, \quad 273\,\mathrm{K} \le T \le 643\,\mathrm{K}$$

Therefore we can plot diffusivity at different temperatures against their corresponding ratios. The linear relationship is again seen, yet an apparent transition appears in the middle. The uncurling speed, which is approximated by Fick's law, also has a similar linear trend, but with no transition involved. Rising temperature also increases peak curvature, which indicates yet another limitation of the theory. Curling speed is also increased at high temperature, indicating the softening of paper.

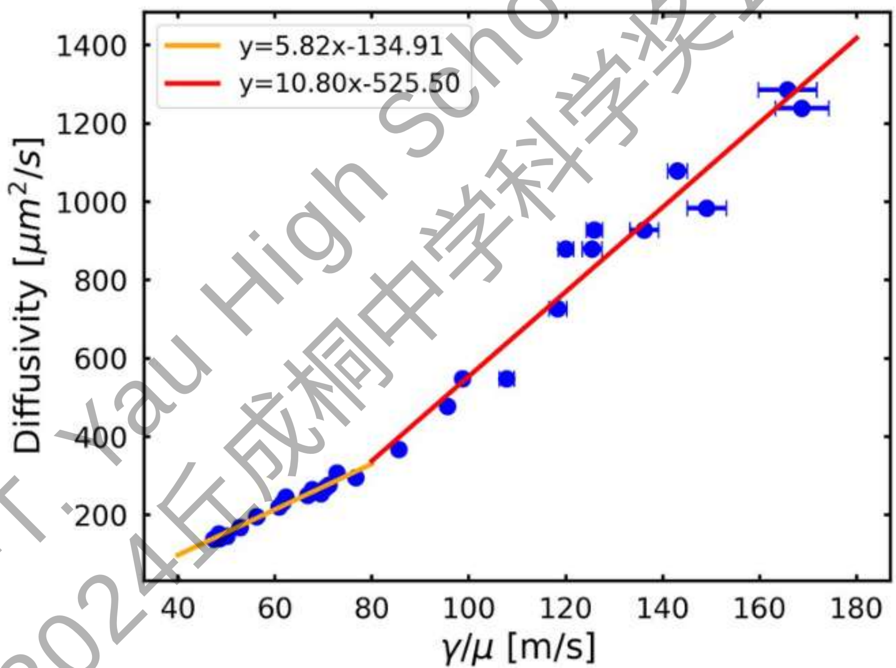


Figure 17: Linear relationship of diffusivity, obtained by peak time, between ratio of surface tension and viscosity, with a transition.

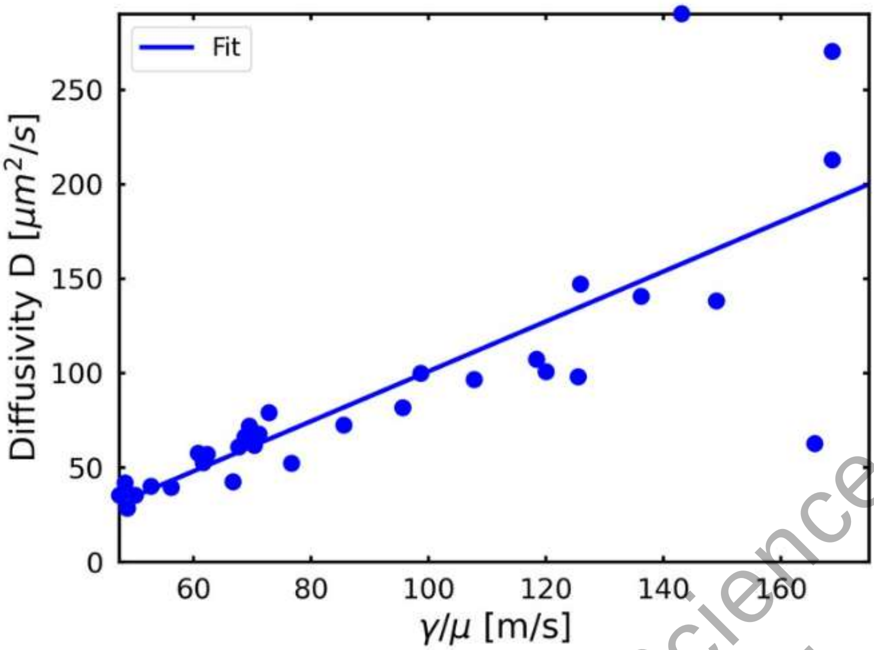


Figure 18: Linear relationship between diffusivity, obtained by uncurling speed, and ratio of surface tension and viscosity.

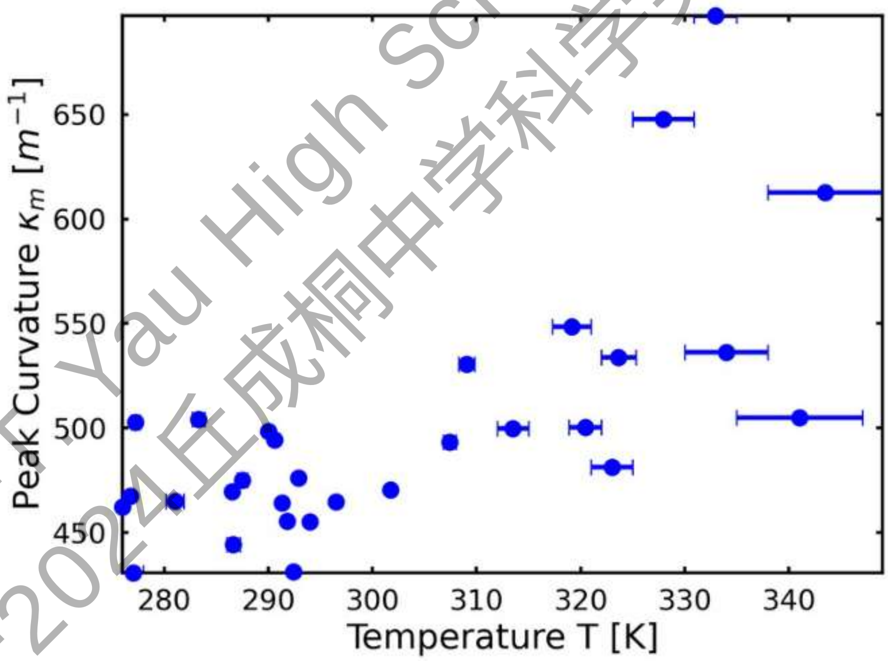


Figure 19: Relationship between peak curvature and temperature, measured in Kelvins.

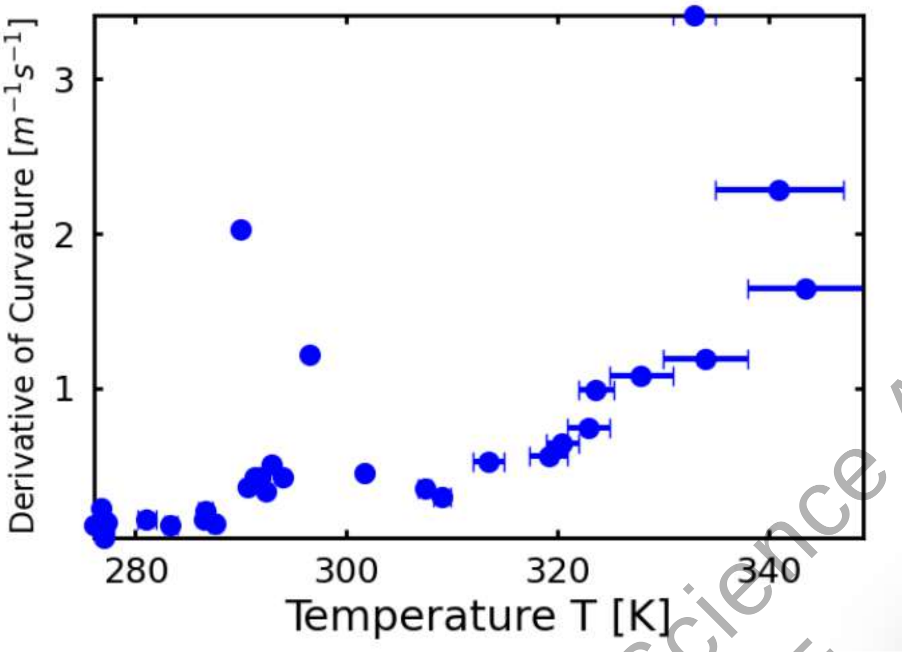


Figure 20: Relationship between curling speed and temperature. Curling speed obtained by averaging the derivative of curvature from start to peak time.

Moreover, it is observed that at high temperatures (50 degrees Celsius) the tracing paper will not uncurl to zero curvature. By setting a flux boundary condition on top in the simulation, it is seen that a constant water content difference in the top and bottom half layer of paper is reached at late time, and therefore curvature approaches a constant. The precise value of the evaporation rate is measured and fitted with an exponential function. It is well-known that the steady-state solution of Fick's law with flux boundary conditions is linear in space, and by this distribution, the final value of curvature can be proven to be:

$$\kappa_{\text{final}} = \frac{\epsilon_{\text{sat}} q}{\theta_{\text{sat}} A D_0}, \tag{28}$$

which shows a linear dependence on evaporation rate. However, the experimental results show a clear transition. We think, along with the transition previously seen, that it is related to the pore depth viewed from the top. The evaporation rate needs to be high enough for the water molecules to escape the deep pores. This fact is supported by comparing the slope of the two lines. In high evaporation rates, the slope is closer to the theory prediction, indicating less obstruction.

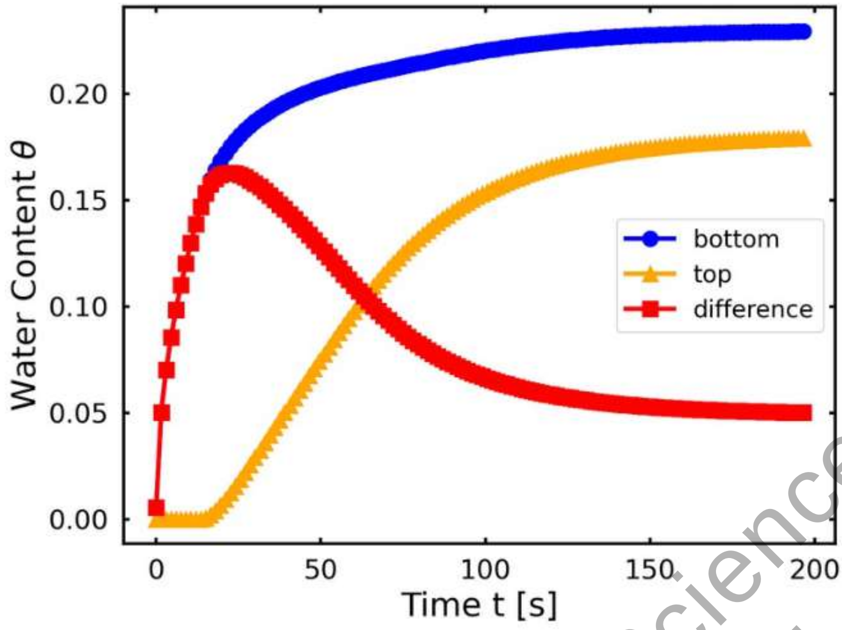


Figure 21: Curve of water content plotted against time. Simulated using $h = 87.15\,\mu\text{m}$, $\frac{q}{AD_0} = 0.01\,\mu\text{m}$, $\theta_{\text{sat}} = 0.25$. Water content of top and bottom are averages of upper half and lower half part of paper.

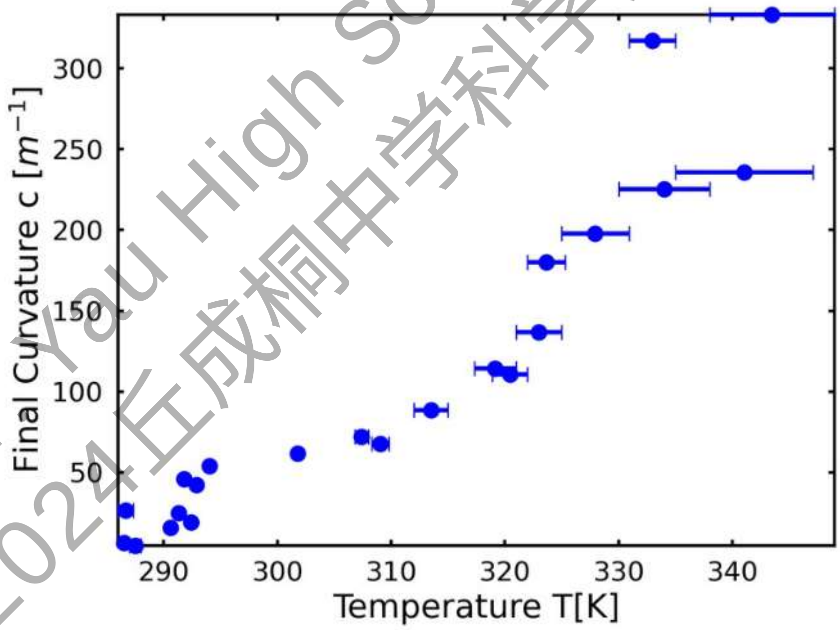


Figure 22: Relationship between final curvature and temperature.

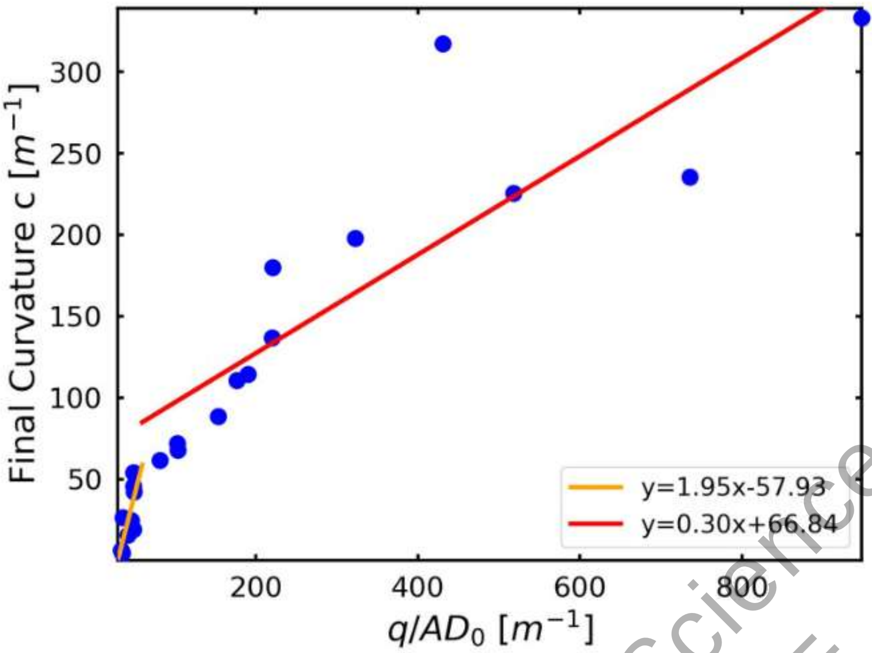


Figure 23: Relationship of final curvature under different temperatures and their corresponding ratio of surface tension and viscosity.

3. Discussion-Salt Concentration

Salt concentration changes both viscosity and surface tension, and by assuming cylindrical pores, formula (5), we see that the diffusivity is linearly related to the ratio $\frac{\gamma}{\mu}$. The experimental data also supports this fact. Further investigations in contact angle may reveal more about the average pore size and pore distribution, and how they can fix the cylindrical pore formula, which is clearly not the case for $n = 1.1$. It should be noted that the model only predicts the peak curvature is determined by expansion and thickness alone. Nevertheless, increasing salt concentration also increases the peak curvature. This cannot be explained by our theory alone, since salt concentration does not affect the maximum expansion, and thickness is controlled throughout the experiment.

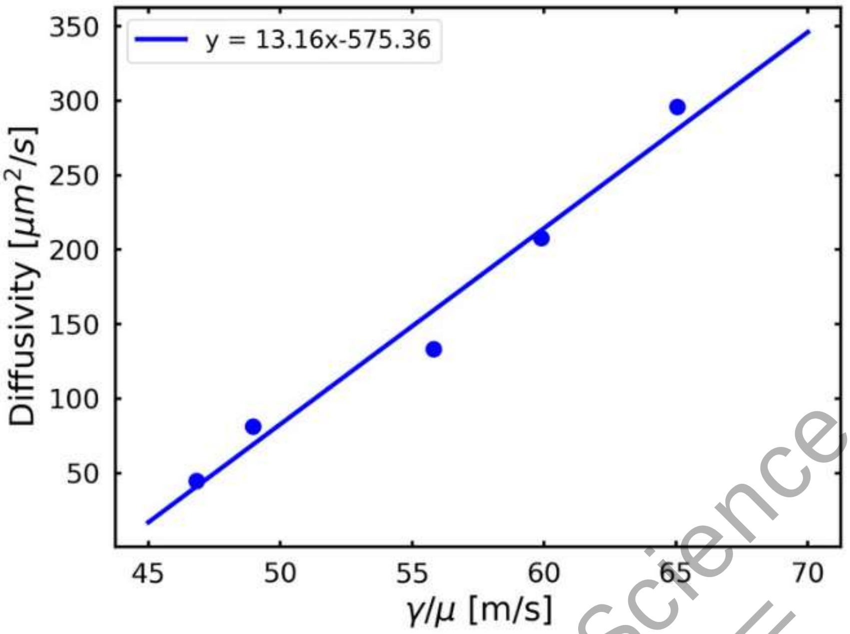


Figure 24: Linear relationship between diffusivity and ratio of surface tension and viscosity. This supports capillary theory, yet the description of slope need further improvements.

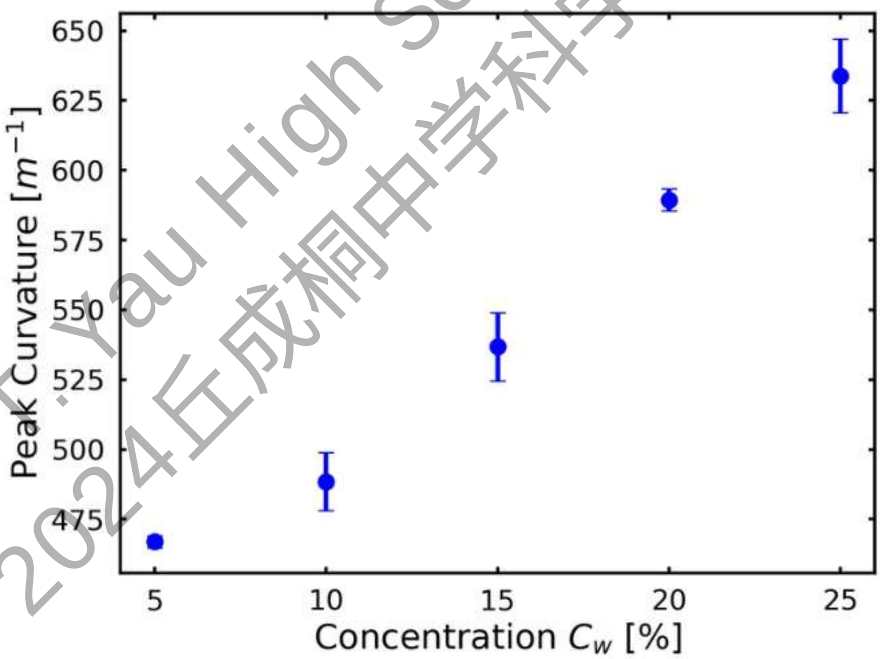


Figure 25: Peak curvature increases with salt concentration. This shows a limitation of the theory, which predicts little change in peak curvature.

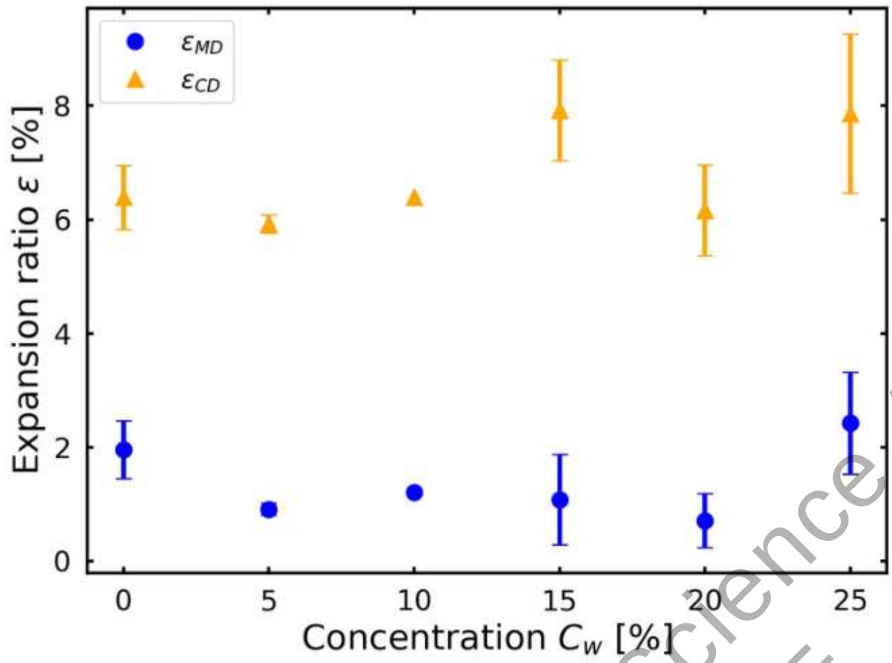
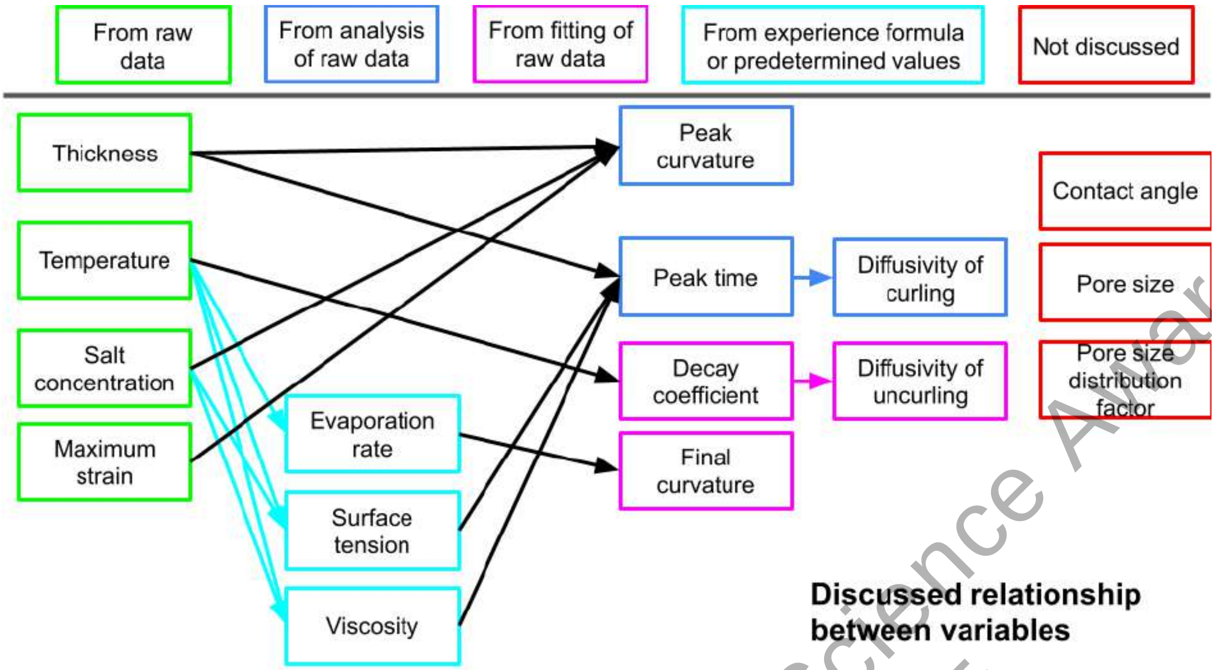


Figure 26: Peak curvature change in salt concentration case is not due to expansion, since saturated expansion rate does not change with salt concentration.

4. Conclusions

Tracing paper will curl, reach a peak curvature, and uncurl when placed gently on water. The geometry of curling includes the curling direction, how many cylinders it produces and whether it will sink, and these properties are affected by anisotropy of expansion, release angle and dimension. The dynamics of curling includes the peak time, peak curvature and final curvature. Surfactant molecules cannot penetrate cellulose surface, so they do not affect the phenomenon. Peak time is affected by thickness of paper and the ratio of surface tension and viscosity. Peak curvature is determined by thickness of paper, which can be predicted by theory, and salt concentration and temperature, which needs further study. Final curvature shows dependence on evaporation, which is not considered in previous study.



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