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论文题目：Mathematical modeling for physician scheduling in fever clinics based on dynamic mixed integer programming

Mathematical modeling for physician scheduling in fever clinics based on dynamic mixed integer programming

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Abstract

Fevers is high morbidity and high risk of cross-infection in hospital because pandemics occur occasionally and spread worldwide. However, capacity of fever clinic is limited. Therefore, it is very important to modeling for physician scheduling with a flexible policy of shifts physicians from other departments when the number of patients exceeds the capacity.

In this paper, a novel mathematical model C3 is proposed to minimize the waiting time cost of fever patients so to avoid cross-infection, which is not found in other literatures. The mathematical model for physician scheduling was established and involved with three parts as work time cost of on-duty physicians (C1), work time cost of secondment physicians from other departments (C2), and C3. Based on logic-based bender decomposition, the mathematical model has been decomposed master problem of physicians staffing and subproblem of physicians scheduling. The subproblem of LBBD decomposed was divided into two-stage optimization problems by column generation method. The number of shifts physicians reduced using brand and bound method to improve bound constraint. The waiting time of fever patients was decreases by optimized service rate and quantity of physicians. Finally, the validity of the mathematical model was verified by real case study in the children hospital in Shanghai. The optimize state value function and optimal policy are implemented in Matlab based on Bellman equation. It can significantly improve the physician scheduling to decrease patients waiting time in fever clinics.

Keywords

Mathematical modeling, Physician scheduling, Fever clinic, Logic-based bender decomposition, Column generation, State value function

Contents

1. Introduction

Fevers is one of the largest morbidity afflicting in hospital. Some virus causes fever. Pandemics still have threatened people, such as the SARS virus in 2003, the swine flu in 2009 [1]. In early 2020, the COVID-19 virus fast spread across the world. There are more 1.18 million COVID-related deaths in the U.S [2]. World Health Organization (WHO) collected the total deaths attributed to COVID-19 is about 7.10 million through official communications under the International Health Regulations on Jun 15, 2025 [3]. During the January in U.S, still 2,861 COVID-related deaths reported in the country on February 1, 2025 [4]. Seasonal influenza easily induced respiratory disease or cardiovascular disease and kills million people each year all over the world. Based on data from October 1, 2024, through May 17, 2025, preliminary estimates of flu medical visits have about 21 million – 37 million, and flu deaths have about 27,000 – 130,000 in the United States [5].

The health care system faces continued challenges for government during pandemic. If virus outbreak, how to manage the influx of patients become challenges for many hospitals were ill-equipped and inadequate medical. To control pandemic requires public health services have adapted to cope with COVID-19, flu, or novel disease outbreaks again. Physicians are the most valuable resources in hospitals and optimizing scheduling to set minimum physician number by the maximum system state and assign a minimum number of physicians to each shift. Therefore, mathematical model for staffing scheduling is important to minimize physicians, and flexibility and shifts staff are more important than before.

In this study, to minimize waiting time of patients, improve capable management of emerging disease outbreaks, we improved a mathematical model based on queueing theory and dynamic mixed integer programming (MIP) to optimize physician scheduling. The number of shifts physician will be reduced using Benders algorithm and column generation to improve bound constraint. Finally, case study is to minimize total costs, and optimize state value function and optimal policy are implemented in Matlab based on on-site data of the large children hospital in Shanghai, and the validity of the model was verified. It can significantly improve the physician scheduling to decrease patients waiting time in fever clinics. The Matlab code as attached in Appendix 1 to share those need to optimize physician scheduling.

2. Literature Review

Responding to disease outbreaks, health services management and planning are related to optimized schedule of physicians. Bowers M.R. et.al. formulated and solved the scheduling optimizing as a binary mixed integer programming in which objective is to minimize on-duty for physicians [6]. Mansini R. and Zanotti R. proposed two mathematical formulations for minimizing total work hours of all the physicians, and designed a heuristic repair method based on extended adaptive large neighborhood search. The proposed algorithms have proved extremely effective by the studied real case [7]. Camiat F. developed a model based mathematical programming to predict the demand of physician workforce with data between 2008 and 2017 from a hospital. The results show that overcrowding relieved after incorporating the work efficiency of physicians in the scheduling process [8]. Huang Y. C. et al. adopted integer programming to optimize the most satisfactory solution on the financial revenue and physicians' schedules with an optimal shift schedule at a hospital in Taiwan [9].

To solve the large scales problem instances and optimize physician scheduling, a continuous linear programming model and a randomized adaptive search procedure have proposed by Cildoz M. et al. Combining heuristic and exact methods, the hybrid algorithm is better than the Integer Linear Programming method [10]. Zaerpour F. et al. formulated the physician scheduling with time-varying productivity limited in a two-stage stochastic programming with L-shaped method and sample average approximation [11].

We occasionally faced critical pandemics, such as COVID, flu. The demand for medical services increases and leads to optimize physician scheduling becomes more important. Liu R. et al. optimized a staffing MIP model for fever clinic with two-phase solution approach and branch-price-and-cut algorithm in COVID-19 pandemic. The method results in reducing queue length and shortening patient waiting time, and minimizing total working hours [12]. Lan S.W. et. al. proposed an integer programming model and explored a variable neighborhood search algorithm with an adaptive heuristic for an integrated operating room and physician scheduling [13]. They proposed an adaptive heuristic dynamic programming to minimize operation. Fugener A. et al. formulated the value of flexible shift physician scheduling in hospitals based on planning for overtime [14]. Liu R. et al. proposed a MIP as time-varying queue with returns for physician scheduling problem [15]. They designed a two-stage optimization algorithm with acceleration strategies to solve the physician staff problem based on machine learning models, and a branch-and-price algorithm to determine the physician scheduling.

The aforementioned literatures review solve physician scheduling problem in same department without considering shifts of physician from different departments during pandemic. In this paper, the mathematical model was established to solve hysician scheduling and shifts with column generation algorithm in pandemic department, respiratory department, emergency department and ICU department. To prove the proposed model, a case was studied to analyze the real world data of one hospital in Shanghai. The minimum value of total physician worktime, the total cost of physician shifts from other departments, and the patients waiting time can quickly obtain under constraint conditions of the hospital.

3. Problem Description and Mathematical Model

There are still possible occur outbreaks of virus diseases around the world. Hospitals need flexible and optimal physician scheduling to shift the physician to serve in the fever clinics from other departments. In this section, the problem description and mathematical model have given for optimizing total time cost (including work time of physicians and waiting time of patients) in fever clinics.

3.1 Definition of variables

Name of set	**Definition**
i/I	index, physicians set, $I = M_1 + M_2$
d/D	index, days set
p/P	index, departments set
s/S(z)	index, shift type set in department z

Parameter	**Definition**
C	the serviced physicians
M_1	the physicians in fever clinics
M_2	the number of secondment physicians in other departments
$h_{d,m}$	which takes 1 if m th physician of other departments is seconded and 0 otherwise.
K	the finite capacity of patients
λ_t	number of patient arrivals in slot t.
P_0	probability that zero patient has to wait in system
P_n	probability that n patients have to wait in system
P_q	probability arriving patient need wait
L_q	average number of waiting patients
L_s	average number of patients
W_q	average waiting time of patients in queue
W_s	average time of patients in the system
Δ	the slot duration
n	batch index
N_t	the total number of batches

3.2 Description of physician scheduling problem

Patients of fever clinics should be modelled with the M/M/C/*K* queueing model are wait for physician service. The first come first served principle for same type patients. All servers of patients are identical, with service rate μ_t .

State transition diagram is shown in Fig. 1. Steady-state probabilities, waiting times, queue lengths are deduced and calculated based the balance equations of visiting patients in fever clinic.

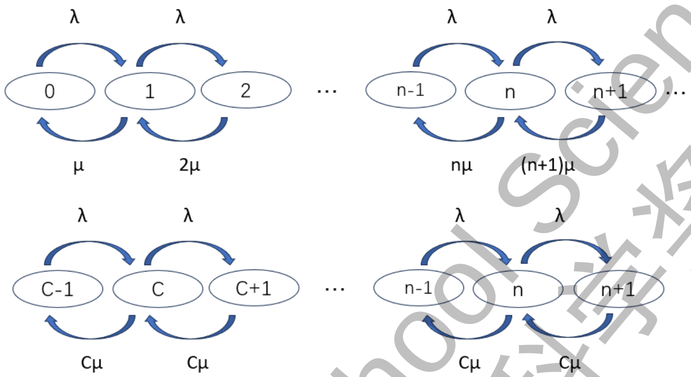

Fig. 1. State transition diagram of visiting patients in fever clinic.

Balance equations as:

$$\begin{cases} \lambda P_0 = \mu P_1 \\ \lambda P_{n-1} + (n+1)\mu P_{n+1} = (\lambda + n\mu) P_n, & \text{for } n < C \\ \lambda P_{n-1} + C\mu P_{n+1} = (\lambda + C\mu) P_n, & \text{for } n \geq C \end{cases} \tag{1}$$

Solution

$$P_n = \begin{cases} \dfrac{(C\rho_c)^n}{n!} P_0, & \text{for } n < C \\ \dfrac{\rho_c{}^n}{C!C^{n-C}} P_0, & \text{for } n \geq C \end{cases} \tag{2}$$

where λ_t is rate of Poisson process of the fever patient arrivals, C means the number of serviced physicians, *n* means the number of patients, $\rho_c = \lambda_t / C\mu_t$ is the average number of patients per minute treated by one physician.

Therefore, the parameters of service process of physicians can be described as:

$$L_q = \sum_{n=C+1}^{\infty} (n-C) \mathrm{P}_n = \frac{(C\rho_c)^C \rho_c}{C!(1-\rho_c)^2} \mathrm{P}_0 \tag{3}$$

$$L_s = \sum_{n=0}^{\infty} n \mathrm{P}_n = \frac{(C\rho_c)^C \rho_c}{C!(1-\rho_c)^2} \mathrm{P}_0 + C\rho_c \tag{4}$$

where, P_0 is probability that zero patient has to wait in system. P_n is probability that n patients have to wait in system.

Following the rules should be met for ensures that a physician either has a day off or has a proper shift, skill, and station assignment.

Remark 1: Fever clinic has day-shift and night-shift. The day-shift is 8:00~17:00, middle shift is 13:00~23:00, and night shift is 23:00~8:00.

Remark 2: In fever clinic or other department, the shifts of physicians should satisfy the following:

① One daily shift is less than two on-duty.

② Each consecutive work time p, $p_{\min} \le p \le p_{\max}$.

③ A physician rest more than p_{gap} hours between two on-duty.

④ The total work time of each physician is less than p_{total}.

Remark 3: In this paper, if pandemic broke out and result in a large number of fever patients, physicians can be shift from pandemic department, respiratory department, emergency department and ICU department. Every department has its own 24-hour regular shift to ensure still provide health care service, in addition to the fever clinic shift.

3.3 Mathematical modeling

Considering the physicians assignment in every scheduling day, we used MIP model and choiced decision variables. Assuming the outpatient clinic has I physicians, with D days scheduling period.

First, we introduce the **decision variables:**

- $V_{i,d,j}$: Binary variable, takes 1 if slot t on-duty, and 0 otherwise.

- $S_{t,d}$: Number of available physicians, in slot t, day d, $S_{t,d} = \sum_{i \in I \cup I'} \sum_{j \in J} V_{i,d,j}$.

- $AWW\left(S_{1,1}, \cdots, S_{t,d}\right)$: Average waiting time of patients.

- $TWW\left(S_{1,1}, \cdots, S_{t,d}\right)$: Total waiting time of patients.

- $QWL\left(L_{t,d-1}, \lambda_{t,d}, s_{t,d}\right)$: The average queue length of patients.

- WL_t^{Start} : The number of patients in the queue in slot t starts.

- WS_t^n : The service time of patients in batch n of the slot t.

- $\theta_{j,t}$: Binary parameter, for each shift j (J=0, 1, 2, 3 mean day off, morning shift, middle shift, night shift, respectively), if t is on-duty then takes 1, otherwise takes 0.

- $P_{m,t,d}$: Preference indicator of physician, which takes 1 if physicians of others departments willing to serve in fever clinics and 0 otherwise.

- $W_{i,t,d}$: The conflict cost if physician i is on duty.

- $L_{t,d}$: The expected queue length.

There are physicians of pandemic department and three available departments (respiratory department, emergency department and ICU department) in fever clinics during pandemics in one hospital at Shanghai. We established the physician staffing model involves objectives as:

***C*1:** The total work time cost of physicians in pandemic department, the working cost of physician is η_1 for a period.

***C*2:** The total secondment work time cost of physicians in other three departments, the secondment cost of physician is η_2 for a period.

***C*3:** The total cost of patients waiting time, the cost coefficient is η_3, per patient, per unit time.

Therefore, the period physician staffing model can be described improved formulated as follows: [16,17]

$$\min\left(C_1 + C_2 + C_3\right) \tag{5}$$

Subject to:

$$C_1 = \sum_{t\in T}\sum_{d\in D}\left(\eta_1 \bullet S_{t,d}\right) \tag{6}$$

$$S_{t,d} = \sum_{i\in I\cup I'}\sum_{j\in J}\left(\theta_{j,t} \bullet V_{i,d,j}\right) \quad t\in T,\ d\in D \tag{7}$$

$$C_2 = \sum_{t\in T, d\in D}\sum_{i=M_1+1}^{M_1+M_2} \eta_2 \bullet h_{d,i} \bullet P_{i,t,d} \tag{8}$$

$$P_{i,t,d} = \sum_{d\in D} V_{i,d,j} \qquad M_1+1 \le i \le M_1+M_2,\ i\in I,\ t\in T \tag{9}$$

$$C_3 = \sum_{\{V_{1,1,1},\cdots,V_{i,t,d}\}} \eta_3 \bullet TWW\left(S_{1,1},\cdots,S_{t,d}\right) \tag{10}$$

$$S_{t,d} \ge 1,\quad t\in T,\ d\in D \tag{11}$$

$$\sum_{j\in J_{d,i}} V_{i,d,j} = 1 \qquad i\in I,\ d\in D \tag{12}$$

$$\sum_{t\in T} V_{i,t,1} \ge 1 \quad i\in I \tag{13}$$

$$V_{i,t+1,1} \geq V_{i,t,2} \quad t \in T \setminus \{|T|\},\ i \in I \tag{14}$$

$$L_{t,d} = QWL\left(L_{t,d-1}, \lambda_{t,d}, s_{t,d}\right) \quad t \in T,\ d \in D \tag{15}$$

where $V_{i,d,j} \in \{0, 1\}$, $\mathrm{L}_{t,d} \geq 0$, $\mathrm{C}_1 \geq 0, \mathrm{C}_2 \geq 0, \mathrm{C}_3 \geq 0$, $S_{t,d} \in N$.

The objective function (5) are considered three parts to minimize the total cost in fever clinics. The three parts include the total cost of work time, the total cost of secondment work time, and total cost of waiting time of patients, respectively. Equation (6) indicates that the total work time cost of physicians can be calculated by adding the work time costs of available physicians in every day. In equation (7), the total number of available on-duty physicians can be calculated. Combining (6) and (7), the total work time cost can be calculated. Equation (8) calculates the total secondment work time cost combined with equation (9) which sums up the preference indicator of physicians willing to shift from other three departments. Equation (10) calculates the total cost of waiting time of patients based on function $TWW\left(S_{1,1}, \cdots, S_{t,d}\right)$ which is described as next paragraph. Constraint (11) ensures that at least one physician working in each room. Equation (12) ensures that physicians involved one shift from other three departments to fever clinic per day. Constraint (13) ensures that every physician at least has one off-day per week. Constraint (14) dictates that a night-shift should be arranged on the next day and followed by an off-day. Equation (15) estimate queue length by function $QWL\left(L_{t,d-1}, \lambda_{t,d}, s_{t,d}\right)$ which described the average queue length of patients in next paragraph.

The function $TWW\left(S_{1,1}, \cdots, S_{t,d}\right)$ in equation (10) and the function $QWL\left(L_{t,d-1}, \lambda_{t,d}, s_{t,d}\right)$ in equation (15) as reported by Hillier and Lieberman [18], which are described as follows:

$$TWW\left(S_{1,1}, \cdots, S_{t,d}\right) = \sum_{n=1}^{N_t} TWW_n\left(S_{1,1}, \cdots, S_{t,d}\right) \tag{16}$$

$$TWW_{N_t}\left(S_{1,1}, \cdots, S_{t,d}\right) = \lambda_t \left(\Delta - \sum_{n=1}^{N_t} WS_t^n\right) * AWW\left(S_{1,1}, \cdots, S_{t,d}\right) \tag{17}$$

$$WS_t^n = \frac{\left(WL_t^{Start} - s_t + 1\right)^{n+}}{s_t \mu - \lambda_t} \tag{18}$$

$$AWW\left(S_{1,1}, \cdots, S_{t,d}\right) = (ac)^{-1} \tag{19}$$

$$QWL\left(L_{t,d-1}, \lambda_{t,d}, s_{t,d}\right) = (ac)^{-1} / \lambda_t \tag{20}$$

$$a = s_t! * \frac{(s_t\mu - \lambda_t)}{(\lambda_t/\mu)^{s_t}} * \sum_{i=0}^{s_t-1} \frac{(\lambda_t/\mu)^i}{i!} + s_t\mu \tag{21}$$

$$c = 1 - \lambda_t / s_t\mu \tag{22}$$

Thus, substituting equations (21) and (22) into (20), the waiting time of patients can be rewritten as:

$$\begin{aligned} AWW(S_{t,d-1}, \lambda_{t,d}, s_{t,d}) &= (ac)^{-1} \\ &= \frac{1}{\left(s_t! * \frac{(s_t\mu - \lambda_t)}{(\lambda_t/\mu)^{s_t}} * \sum_{i=0}^{s_t-1} \frac{(\lambda_t/\mu)^i}{i!} + s_t\mu \right)(1 - \lambda_t/s_t\mu)} \\ &= \frac{s_1\mu}{\left[s_1! * (s_1\mu - \lambda_1) * \sum_{i=0}^{s_1-1} \frac{(\mu/\lambda_t)^{s_1-i}}{i!} + s_1\mu \right](s_1\mu - \lambda_1)} \end{aligned} \tag{23}$$

Let

$$f_1(s_1) = s_1\mu - \lambda_1 \tag{24}$$

$$f_2(s_1) = \sum_{i=0}^{s_1-1} \frac{(\mu/\lambda_t)^{s_1-i}}{i!} \tag{25}$$

$$f_3(s_1) = s_1\mu - \lambda_1 \tag{26}$$

$$AWW(S_{t,d-1}, \lambda_{t,d}, s_{t,d}) = g(s_1) = \frac{s_1\mu}{\left[s_1! * f_1(s_1) * f_2(s_1) + s_1\mu \right] f_3(s_1)} \tag{27}$$

Given that $f_1'(\lambda_1) = -1 < 0$, $f_2'(\lambda_1) = \sum_{i=0}^{s_1-1} \frac{(i - s_1)(\lambda_1/\mu)^{i-s_1-1}}{\mu * i!} < 0$, thus $f_1(\lambda_1)$ and $f_2(\lambda_1)$ are decreasing obviously with λ_1. Because $f_1(\lambda_1) > 0$ and $f_2(\lambda_1) > 0$, thus $g(s_1) = \frac{s_1\mu}{\left[s_1! * f_1(s_1) * f_2(s_1) + s_1\mu \right] f_3(s_1)}$ is increasing with λ_1.

Similarly, if μ is increasing, $g(s_1)$ is decreasing.

Assume $s_{11} > s_{12} > 0$, then.

$$f_1(s_{11}) - f_1(s_{12}) = (s_{11} - s_{12})\mu > 0 \tag{28}$$

$$f_2(s_{11}) - f_2(s_{12}) = \sum_{i=s_{12}+1}^{s_{11}-1} \frac{(\mu/\lambda_t)^{s_{11}-i}}{i!} + \sum_{i=0}^{s_{12}-1} \frac{(\mu/\lambda_t)^{s_{12}-i}}{i!} \left[(\mu/\lambda_t)^{s_{11}-s_{12}} - 1 \right] > 0 \tag{29}$$

$$f_3(s_{11}) - f_3(s_{12}) = (s_{11} - s_{12})\mu > 0 \tag{30}$$

It could obtain that

$$g(s_{11})=\frac{\mu}{\left[(s_{11}-1)!*f_1(s_{11})\ *f_2(s_{11})+\mu\right]f_3(s_{11})} \tag{31}$$

$$g(s_{12})=\frac{\mu}{\left[(s_{12}-1)!*f_1(s_{12})\ *f_2(s_{12})+\mu\right]f_3(s_{12})} \tag{32}$$

Thus, it can deduce that $g(s_{11})<g(s_{12})$, which means that $g(s_1)$ is decreasing with s_1. As for other slot t, the monotonicity of $g(s_t)$ is consistent with $g(s_1)$.

The total cost of waiting time of patients (C_3) can be calculated by equations (10) combining with (16), (17), (18) and (23). According to (27), it can be deduced that the value of C_3 is increasing if λ_1 increasing, and decreasing if μ increasing, and decreasing if s (number of physicians) increasing. It indicates there is at least one minimal value for C_3. As described by Hillier and Lieberman [18], the minimum the patient waiting time target, when TWW has a minimal value about optimal φ_t, s_t has a minimal value for C_3.

To optimizing objective function (5), we adopted linearization as reported by Wang et al [16]. The capacity of patients on duty $O_{t,d}$ can be written as:

$$O_{t,d}=\mu\cdot S_{t,d}\cdot\Delta\cdot\rho_{t,d} \tag{33}$$

Given the capacity of queue length $L_{t-1,d}$ can be written as:

$$L_{t-1,d}+\lambda_{t,d}=L_{t,d}+O_{t,d} \tag{34}$$

where $\mu\cdot S_{t,d}\cdot\Delta$ is service capacity, $\rho_{t,d}$ is utilization rate.

Equation (33) calculates the departures of patients in period t. Equation (34) shows the number of patients at time t equals the number of patients at time t-1, adding the arrived patients from t-1 to t, and subtracting the departures of patients during time t-1 to t.

In addition, the variable $y_{t,d,i}$ takes 1 if there are i on-duty physicians in period t,d, and 0 otherwise. Thus

$$\sum_{i=1}^{M_1+M_2} i\bullet y_{t,d,i}=S_{t,d} \quad t\in T,\ d\in D \tag{35}$$

$$\sum_{i=1}^{M_1+M_2} y_{t,d,i}=1 \quad t\in T,\ d\in D \tag{36}$$

Therefore, equations (33) can be rewritten as:

$$O_{t,d} \le \mu \cdot i \cdot \Delta \cdot \rho_{t,d} + \text{big}M \cdot \left(1 - y_{t,d,i}\right) \tag{37}$$

$$O_{t,d} \ge \mu \cdot i \cdot \Delta \cdot \rho_{t,d} - \text{big}M \cdot \left(1 - y_{t,d,i}\right) \tag{38}$$

where big$M$ is a enough large positive constant.

To minimize the queue length, the approximate function can be written as:

$$L_{t,d} \ge k_{i,n}\left(\rho_{t,d} + y_{t,d,i} - 1\right) + y_{t,d,i} \cdot b_{i,n} \quad 1 \le n \le N,\ 1 \le i \le M_1 + M_2 \tag{39}$$

where $k_{i,n}$ and $b_{i,n}$ are secant lines [16].

Logic-based Bender decomposition (LBBD): it is applied to deal with the physicians staffing and scheduling problem [16]. The LBBD master problem calculated the number of on-day physicians to ensure the queue length $L_{t,d} < \overline{L}_{t,d}$. The physician staffing model is presented as objective function (5).

$$\min\ \left(\hat{C}_1 + \hat{C}_2 + C_3\right) \tag{40}$$

Subject to: constrains (11) ~ (14), (34), (37) ~ (39),

and $y_{t,d,i} \in \{0, 1\}$, $\rho_{t,d} \in (0, 1)$, $L_{t,d} \ge 0$, $S_{t,d} \in N$, $1 \le d \le 7$, $1 \le t \le 24$,

$M_1 + 1 \le i \le M_1 + M_2$, $i \in I\left(I = M_1 + M_2\right)$.

To satisfy the physicians requirement every day, the number of on-day physicians for each period $\hat{S}_{t,d}$ should obtain in each iteration. The work time cost of physicians can be described as $\hat{C}_1$ and $\hat{C}_2$ for over-time queue length.

The LBBD subproblem $\hat{C}_3$ allocates physician scheduling to min the total cost of waiting time of patients. The objective function (5) can be rewritten as:

$$\min\ \left(\dot{C}_1 + \dot{C}_2 + \hat{C}_3\right) \tag{41}$$

where $\dot{C}_1 = \hat{C}_1$, $\dot{C}_2 = \hat{C}_2$.

subject to:

$$\dot{C}_1 = \hat{C}_1 = \sum_{t \in T} \sum_{d \in D} \sum_{i \in I \cup I'} \sum_{j \in J} \left(\eta_1 \bullet \theta_{j,t} \bullet V_{i,d,j}\right) \tag{42}$$

$$\sum_{i \in I \cup I'} \sum_{j \in J} \left(\theta_{j,t} \bullet V_{i,d,j}\right) \ge \hat{S}_{t,d} \quad 1 \le t \le 24,\ 1 \le d \le 7,\ I = M_1 + M_2 \tag{43}$$

Equations (43) indicates that the number of on-duty physicians should be more than $\hat{S}_{t,d}$. Since subproblem $\hat{C}_3$ are not linear programming problem, the LBBD cut cannot be generated by pairwise change. As Subproblem $\hat{C}_3$ are sets cover problem, the cuts are proposed to reflect the information of the subproblem's results to the master problem. The optimizing $\hat{C}_1 + \hat{C}_2$ can be iterated by

$$\left(\hat{C}_1+\hat{C}_2\right)\geq\left(C_1+C_2\right)\bullet\left[1-\sum_{t=1}^{24}\sum_{d=1}^{7}\left(1-z_{t,d}\right)\right] \tag{44}$$

$$z_{t,d}\bullet\text{big}\left(M_1+M_2\right)\geq s_{t,d}-\left(\hat{s}_{t,d}-1\right) \tag{45}$$

$$\left(z_{t,d}-1\right)\bullet\text{big}\left(M_1+M_2\right)\leq s_{t,d}-\left(\hat{s}_{t,d}-1\right) \tag{46}$$

where $z_{t,d}$ is an auxiliary variable (0 or 1), which takes 1 if $s_{t,d}\geq\hat{s}_{t,d}$ and 0 otherwise.

Equations (45) and (46) indicate that if $s_{t,d}\geq\hat{s}_{t,d}$, then $C_1+C_2\geq\hat{s}_{t,d}$. It means the minimum on-duty and shifts cost of physicians is more than $\hat{s}_{t,d}$ in each period. Therefore, the number of physicians needed is more than $\hat{s}_{t,d}$ in every period, the total on-duty and shifts costs should more than $\hat{C}_1+\hat{C}_2$.

3.4 Optimizing physician scheduling

Branch and bound algorithm (BB): the physician scheduling is challenging MIP, combinatorial optimization algorithms need to be developed. To solve the variables to integers, BB is applied to quickly calculate integer solutions. At the nodes where branching is required, the three variables (*i, d, j*) - the doctor, the number of days, and the time period need to satisfy

$$0<\sum_{j=0}^{3}\left(\theta_{j,t}\bullet V_{i,d,j}\right)<1 \tag{47}$$

Hence, if $\sum_{j=0}^{3}\left(\theta_{j,t}\bullet V_{i,d,j}\right)\leq 0$, it means physician is day off. On the contrary, if $\sum_{j=0}^{3}\left(\theta_{j,t}\bullet V_{i,d,j}\right)\geq 1$, it means physician is be on duty.

The physician scheduling problem have a large number of shifts can be scheduled. Thus, the subproblem C_1, C_2, C_3 more challenging.

Column generation (CG): it is used in the optimal scheduling solution as reported by Wang C. K. et al [16]. In the CG method, use linear programming to solve physician scheduling problem (subproblem of LBBD decomposed), which is decomposed into the restricted master problem (RMP) and the pricing problem [19]. The RMP assigns physicians to daily on-duty for patients, and the pricing of hospital services determines whether a particular daily on-duty shift for patients. To solve the master problem of physician staffing by iterating, let define $J_{d,i}$ as the set of daily shifts of physician. Therefore, objective function (40) can be rewritten as:

$$
\begin{aligned}
\min & \left(\hat{C}_1+\hat{C}_2+C_3\right) \\
&=\sum_{t\in T}\sum_{d\in D}\sum_{i\in I\cup I'}\sum_{j\in J_{d,i}}\left(\eta_1\bullet\theta_{j,t}\bullet V_{i,d,j}\right)+C_2\bullet\left[1-\sum_{t=1}^{24}\sum_{d=1}^{7}\left(1-z_{t,d}\right)\right] \\
&\quad+\sum_{\{V_{1,1,1},\cdots,V_{i,t,d}\}}\eta_3\bullet\lambda_t\left(\Delta-\sum_{n=1}^{N_t}WS_2^n\right)*AWW\left(S_{1,1},\cdots,S_{t,d}\right) \\
&=\sum_{t\in T}\sum_{d\in D}\sum_{i\in I\cup I'}\sum_{j\in J_{d,i}}\left(\eta_1\bullet\theta_{j,t}\bullet V_{i,d,j}\right)+\sum_{t\in T,d\in D}\sum_{m=M_1+1}^{M_1+M_2}\eta_2\bullet h_{d,m}\bullet P_{m,t,d}\bullet\left[1-\sum_{t=1}^{24}\sum_{d=1}^{7}\left(1-z_{t,d}\right)\right] \\
&\quad+\sum_{\{V_{1,1,1},\cdots,V_{i,t,d}\}}\eta_3\bullet TWW\left(S_{1,1},\cdots,S_{t,d}\right)
\end{aligned}
\tag{48}
$$

subject to:

$$\sum_{i\in I}\sum_{j\in J_{d,i}}\left(\theta_{j,t}\bullet V_{i,d,j}\right)\geq\hat{S}_{t,d}\quad d\in D,\ t\in T \tag{49}$$

$$\sum_{d\in D}V_{i,d,j}\geq 1\qquad t\in T \tag{50}$$

$$\sum_{t\in T}V_{i,d,j}\leq 1\qquad i\in I',\ d\in D \tag{51}$$

$$V_{i,d+1,1}\geq V_{i,d,2}\qquad i\in I,\ d\in D\setminus\{|D|\} \tag{52}$$

$$0\leq V_{i,d,j}\leq 1\qquad i\in I\cup I',\ t\in T,\ d\in D,\ j\in J_{d,i} \tag{53}$$

As reported by Wang C. K. et al [16], the set of $J_{d,i}$ contain day off shift, night-shift, and whole day working shift. An restricted master problem equivalent to several column generation subproblems donate as CG-SP$_{i,d}$, which means each subproblem generates a set of valid shifts for physician i.

Defining the decision variables, a binary variable e_t, which takes 1 if shift covers period t, and 0 otherwise. The objective of restricted master problem is to minimize the number of constraints corresponding to the shift schedule. Thus, the reduced cost of the restricted master problem rewritten as:

$$
\begin{cases}
\sum_{t=1}^{24}\left(\Delta\cdot P^{\text{work}}\cdot e_t-\beta_{d,t}e_t\right)-\gamma_{d,i} & 1\leq i\leq M_1 \\
\sum_{t=1}^{24}\left(\Delta\cdot P^{\text{work}}\cdot e_t-\beta_{d,t}e_t\right)-\gamma'_{d,i}+\eta_2 & M_1+1\leq i\leq M_1+M_2
\end{cases}
\tag{54}
$$

where $\beta_{d,t}$ is the dual cost of constraint (49), $\gamma_{d,i}$ is the dual cost of constraint (13), $\gamma'_{d,i}$ is the dual price of constraint (51). Since the $J_{d,i}$ defined as the set of daily shifts, thus, it has $e_t=0,\ \forall\ 17\leq t\leq 24$.

Thus, minimizing equation (54) means to minimize the following item:

$$\min \sum_{t=1}^{16}\left(\Delta\cdot P^{\text{work}}\cdot e_t-\beta_{d,t}e_t\right) \tag{55}$$

Thus, the CG-SP$_{i,d}$ should be regarded as a shortest path problem with restricted constraints. The equation (55) indicates that the contribution of shift is $e_t=1$, the contribution for objective function is $\Delta\cdot P^{\text{work}}\cdot e_t$. Between the period containing beginning, shifts and on-duty, t the Bellman equation can be written as optimal target of the value function $v(u, j, r)$, following as:

$$v(u,j,r)=\min\left\{v(u+1,j,r),\ \min_{p_{\min}\le p\le\min(p_{\max},r)}\left[v\left(u+p+p_{gap},j-1,r-p\right)+\sum_{t=u}^{u+p-1}\left(\Delta\cdot P^{\text{work}}-\beta_{d,t}\right)\right]\right\} \tag{56}$$

If u without as the start of a new shift, the $v(u+1, j, r)$ is optimal target of Bellman equation in the (56). Conversely, if choosing u as the start of a new shift, where

$$\min_{p_{\min}\le p\le\min(p_{\max},r)}\left[v\left(u+p+p_{gap},j-1,r-p\right)+\sum_{t=u}^{u+p-1}\left(\Delta\cdot P^{\text{work}}-\beta_{d,t}\right)\right] \tag{57}$$

Equation (57) is the optimal target of Bellman equation (56).

Hence, minimizing the item of equation (55), it means minimize the cost which can be calculated the optimal target as:

$$\min_{1\le r\le p_{total}}\left\{v(1,1,r),\ v(1,2,r)\right\} \tag{58}$$

The solution of equation (58) is the optimal physician scheduling provided by CG-SP$_{i,d}$ which should be applied to the shift set $J_{i,d}$. The CG algorithm will iterate untill there no physician shift with a negative cost. The solution of CG-RMP is optimal value of linearized LBBD subproblem.

4. Implementation

In this section, we implement the mathematical model for physician scheduling to generate shift schedules using two on-site data of hospital and evaluate the mathematics mode. All data of two test examples are obtained from the fever clinic of two large hospitals in Shanghai during the summer, respectively. All numerical calculations are run by using Matlab.

In the fever clinic of one large adult hospital in Shanghai, the working hours of physicians are 8:00-12:00, and 13:30-17:00 on the day shift. Patients are numbered according to their arrival time and wait in one queue for physician service. Patients serviced according to the first come first served principle. With 2 physicians, the detailed data of visiting patients are record on-site from 8:00 to 12:00. on July 16th, 2025, which shown in Table 1. The data show patients 7# and 13# did not appear in this table because they may be changed to others clinic. In this table, there are 2 physicians, but only 15 fever patients. Hence, the fever patients did not wait, and the physician was not busy. The hospital should arrange only one physician on duty in fever clinic during without pandemic.

Table 1

Visiting patients of fever clinic in the large adult hospital in Shanghai (July 16th, 2025)

Time	Physician 1# (patients No)	Physician 2# (patients No)	Time	Physician 1# (patients No)	Physician 2# (patients No)
8:00	1#	2#	10:00		
8:10			10:10		11#
8:20	3#		10:20		
8:30		4#	10:30	12#	
8:40	5#		10:40		
8:50			10:50		14#
9:00		6#	11:00	15#	
9:10	8#		11:10		
9:20			11:20		16#
9:30		9#	11:30	17#	
9:40			11:40		
9:50	10#		11:50		
10:00			12:00		

Moreover, we investigated another fever clinic in other large adult hospital in Shanghai on August 18th, 2025. Similarly, according the detailed data from on-site at different large adult hospital in Shanghai, the data show that only few fever patients. Therefore, a hospital shall have one physician on duty in fever clinic, and more physicians (from fever clinic and other clinics) call at all times and available to the hospital on-site during pandemic.

There are few fever patients in the large adult hospital in Shanghai during the summer. Therefore, we investigated fever clinic in the large children hospital in Shanghai. Patients are numbered according to the time of their arrival and wait in one queue. The detailed data of visiting patients are record on-site from 8:00 to 12:20. on August 26th, 2025, which shown in Table 3. It shows that 2 physicians and 2 experts on duty in the fever clinic in the day. In the physicians 1# and 2# clinic, the data show

128 patients have received treatment, but patients 4#, 13#, 23# et. al. (19 patients) did not appear in this table because they may be changed to others clinic. Meanwhile, in the experts 1# and 2# clinic, the data show 42 patients have received treatment, but patients 4#, 10#, 14# et. al. (10 patients) did not appear in this table because they may be changed to others clinic. Table 2 shows that patients need wait long time in general fever clinic from 8:00 a.m. to 9:00 a.m. because many patients queuing up. In the experts 1# and 2# clinic, because the number of arrival patients is limited and the cost is relatively high, so the queuing patients need not wait long time.

Table 2

Visiting patients of fever clinic in the children hospital in Shanghai (August 26th, 2025)

Time	Physician 1# (patients No)	Physician 2# (patients No)	Expert 1# (patients No)	Expert 2# (patients No)
8:00	1#,2#,6# 8#,10#,12#	3#,5#,7#,9# 11#,13#,15#	1#,3#	2#,5#
8:10	16#,18#,19#,21# 24#,27#,29#	17#,20#,22# 25#,26#,28#	6#,8#	7#,9#
8:20	30#,32#,33# 36#,39#,42#,	31#,35#,37# 38#,41#,45#	11#,15#	12#,13#
8:30	46#,49#,51# 53#,55#,57#	47#,48#,50# 52#,56#,58#	17#,19#	16#,18#
8:40	60#,62#,66# 68#,71#	61#,63#,65# 67#,70#	20#,22#	21#,23#
8:50	73#,74#,76# 79#,81#	72#,75#,77# 78#,80#	26#	25#
9:00	83#	82#,84# 70#(return)		27#, 29#
9:10	85#, 87#	86#	28#	
9:20	81#(return)	88#,89#	28#	31#
9:30	90#, 92#, 94#	91#,93#	36#	
9:40	97#, 98# 83#(return)	78#(return) 99#		38#
9:50	102#	86#(return) 101#		
10:00	85#(return) 92#(return)	67#(return) 75#(return) 103#		
10:10	104#	105#	34#	40#,41#,42#
10:20	108#	91#(return) 93#(return) 109#	43#	

To be continued

Time	Physician 1# (patients No)	Physician 2# (patients No)	Expert 1# (patients No)	Expert 2# (patients No)
10:30	111#,112# 90#(return)	113#	44#,45#	
10:40	118#	101#(return) 119#	32#,39#	
10:50	120#	80#(return) 121#	45#(return)	38#(return)
11:00	111#(return),123# 85#(return)	122#, 124#	34#(return)	
11:20	115#,129#,130#	128#		48#
11:30	118#(return) 120#(return) 131#		34#(return)	41#(return)
11:40	116#,132#, 133#,134#			50#
11:50	137#, 138#	109# (return) 122#(return) 124#(return)		
12:00	131#(return) 140#	113#(return) 128#(return) 141#,142#		
12:10		143#, 144#	43#(return) 49#	51#
12:20		145#,146#,147#	52#	48#(return)

As shown in Table 2, the patients need wait long time in general fever clinic from 8:00 a.m. to 9:00 a.m. There is arriving about 84 fever patients during 60 min. Hence, the average number of patient arrivals is $\lambda_t = 1.4$ per min. Assume that the service time of physician follows a negative exponential distribution with average service rate $\mu = 1.008$. Average number of patients treated is $\rho_c = \frac{\lambda_t}{C\mu} = \frac{1.4}{2\times 1.008} = 0.69$ per minute. Hence, average number of waiting patients is about 11.5 deduced by equation (4). The average waiting time of patients is about 5.96 minutes.

To reduce the waiting time of patients, the fever clinic should temporary shift 2 physicians from pandemic department, respiratory department, emergency department or ICU department from 8:00 to 9:00 every day. If there are 4 physicians to diagnose the fever patients, the average number of patients treated is ρ_c=0.345 per minute.

Hence, average number of waiting patients is about 2.6 deduced by equation (4).

The average waiting time of patients is about 2.51 minutes.

Table 3
Reducing waiting time of visiting patients with more physicians in fever clinic from 8:00 a.m. to 9:00 a.m (August 26th, 2025)

Time	Patients number	Average waiting time with 2 physicians (min)	Average waiting time with 4 physicians (min)
8:00	1#,2#,6#,8#,10#,12# 3#,5#,7#,9#,11#,13#,15#	3.62	1.59
8:10	16#,18#,19#,21#,24#,27#,29# 17#,20#,22#,25#,26#,28#	5.37	2.15
8:20	30#,32#,33#,36#,39#,42#, 31#,35#,37#,38#,41#,45#	6.25	2.72
8:30	46#,49#,51#,53#,55#,57# 47#,48#,50#,52#,56#,58#	6.81	3.05
8:40	60#,62#,66#,68#,71# 61#,63#,65#,67#,70#	7.13	3.11
8:50	73#,74#,76#,79#,81# 72#,75#,77#,78#,80#	7.05	3.08
9:00	83# 82#,84#,70#(return)	6.51	2.63

With more physicians., the results of reducing waiting time of visiting patients are shown in Table 3 in fever clinic from 8:00 a.m. to 9:00 a.m. The results indicate that the waiting time of visiting patients has been greatly reduced based on equation (4). As most of the patients come to the hospital between 8:00 a.m. and 8:30 a.m., therefore, the longest average waiting time of patients is about 7.13 minutes at 8:40. If the number of physicians increased from 2 to 4, the average waiting time of patients have reduced to 3.11 minutes at 8:40. Patients usually earlier arrive at the hospital, they will have to wait long time before 9:00 a.m. Therefore, hospitals need flexible shift physicians from other department to serve in fever clinic to reduce the waiting time of patients from 8:00 a.m. to 9:00 a.m.

To cope with the challenges of outbreak of epidemic or pandemic in Shanghai, the large children hospital has prepared a large room to deal with a large number of fever patients, which can accommodate 9 physicians on duty and can also receive 100 fever patients at the same time. In this summer, there was no outbreak of epidemic or pandemic in Shanghai, so this room is closed.

To optimize physicians scheduling, we are using Matlab tool to solve the optimal value of Bellman equation with recursive Function. Four sets of states constructed as

S=[1, 2, 3, 4], which 1 means that more than 100 fever patients arriving every hour, 2 means that $70 <$ fever patients ≤ 100 arriving every hour, 3 means that $40 <$ fever patients ≤ 70 arriving every hour, 4 means that $20 <$ fever patients ≤ 40 arriving every hour. Moreover, two strategy sets have constructed as A= [1, 2], which 1 means 4 physicians on duty, 2 means 2 physicians on duty. The discount factor is setting 0.5, and convergence threshold value is setting 0.00001. The Matlab code as attached in Appendix 1, and the program run result as follow:

Optimal state value function as:	**Optimal policy** as:
6.7607	1
5.5987	1
4.9610	1
4.3816	2

5. Conclusions

In this paper, a mathematical model of physician scheduling was established which integrated and improved the math models from the articles [16] and [17]. A novel mathematical model C3 is proposed to minimize the waiting time cost of fever patients so to avoid cross-infection，which is not found in other literatures. The objective function C_3 was minimized by applying Bellman equation to optimize state value function and optimal policy. To improve bound constraint of the mathematical model, the objective function has been LBBD decomposed into master problem of staff allocation and subproblem of physician scheduling. The subproblem of LBBD decomposed was divided into two-stage optimization problems by column generation method to obtain column of iteration number and column of solving time to enhance the performance of subproblem.

In this thesis, the validity of the mathematical model has been verified can reduce the waiting time of fever patients based the on-site data of hospital (the large children hospital in Shanghai), by optimizing master problem (state value function) and subproblem (policy). It is found that the waiting time of fever patients can reduced about 57% if the number of physicians from 2 add to 4 from 8:00 a.m. to 9:00 a.m in the children hospital in Shanghai on August 26th, 2025. Physicians should have flexible scheduling and secondment from other clinics to deal with the uncertain fever patients.

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Acknowledgements

I would like to thank my mathematics teacher Shou-Bo Lu for his direct instruction in mathematics theory, and help in guidance on writing thesis. Under the guidance of senior teacher Shou-Bo Lu, the author Ao Wang independently integrated and deduced the mathematical model based on relevant references in this thesis.

I would like to give sincerely thank to Professor Dan Shapiro, with whom I had a lot of discussions with at Ross Mathematics Camp in 2024. He taught me how to approach vast and seemingly unsolvable problems by concretizing abstract concepts to identify and verify underlying patterns.

I would also like to thank Professor Rick Sommer at Stanford University, who lectured me at SUMaC math camp in 2025. Under his instruction, I was able to explore abstract algebra and number theory in greater depth and develop a stronger understanding of their fundamental concepts. The insights and encouragement I received not only improved my overall mathematical ability but also helps me to build interdisciplinary thinking skills. This became an important skill once I set about doing my research.

I hereby sincerely thank all of the people who contributed to this paper, or help me improved my math skills.

Appendix 1: Matlab code to solve the optimal value of Bellman equation

```
% parameter setting
S = [1, 2, 3, 4];    % Set of states
A = [1, 2];    %    Strategy set
gamma = 0.9;    % discount factor
epsilon = 0.00001;    % convergence threshold value

% matrix of transition probability
P = zeros(length(S), length(S), length(A));
P(:,:,1) = [0.8, 0, 0.2, 0.0; 0.7, 0.0, 0.3, 0.0; 0.2, 0.3, 0.5, 0.0; 0.0, 0.0, 0.3, 0.7];
P(:,:,2) = [0.0, 0.1, 0.0, 0.9; 0.0, 0.3, 0.0, 0.7; 0.0, 0.6, 0.4, 0.0; 0.0, 0.4, 0.0, 0.6];

% Reward Matrix
R = zeros(length(S), length(A));
R(:,1) = [1, 0, 0, 0];
R(:,2) = [0, 1, 0, 0];

% Initialize the value function
V = zeros(length(S), 1);

% Value Iteration
while true
    V_prev = V;
    for s = 1:length(S)
        Q_values = zeros(1, length(A));
        for a = 1:length(A)
            Q_values(a) = R(s, a) + gamma * sum(P(s,:,a) .* V');
        end
        V(s) = min(Q_values);
    end

    % Convergence judgment
    if min (abs(V - V_prev)) < epsilon
        break;
    end
end

% Output "Optimal Value Function"
disp('Optimal Value Function:');

disp(V);

% Calculate the optimal strategy
policy = zeros(length(S), 1);
```

```
for s = 1:length(S)
    Q_values = zeros(1, length(A));
    for a = 1:length(A)
        Q_values(a) = R(s, a) + gamma * sum(P(s,:,a) .* V');
    end
    policy(s) = find(Q_values == min(Q_values), 1);
end

% Output “Optimal Policy”
disp('Optimal Policy:');
disp(policy);
```

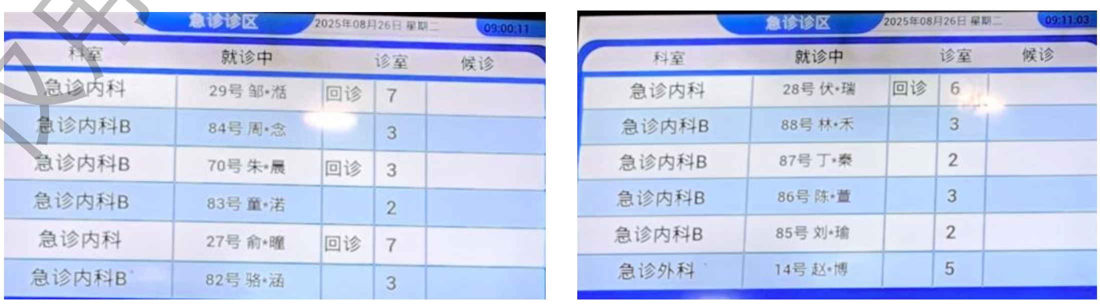
Appendix 2: Hospital site photos.

(a) (b)

Fig. 3. Fever clinic in the hospital on July 16th, 2025. (a) Physician 1# in room 2#. (b) Physician 2# in room 3#.

(a) (b)

Fig. 4. Fever clinic in the hospital on July 16th, 2025. (a) Expert 1# in room 6#. (b) Expert 2# in room 7#.



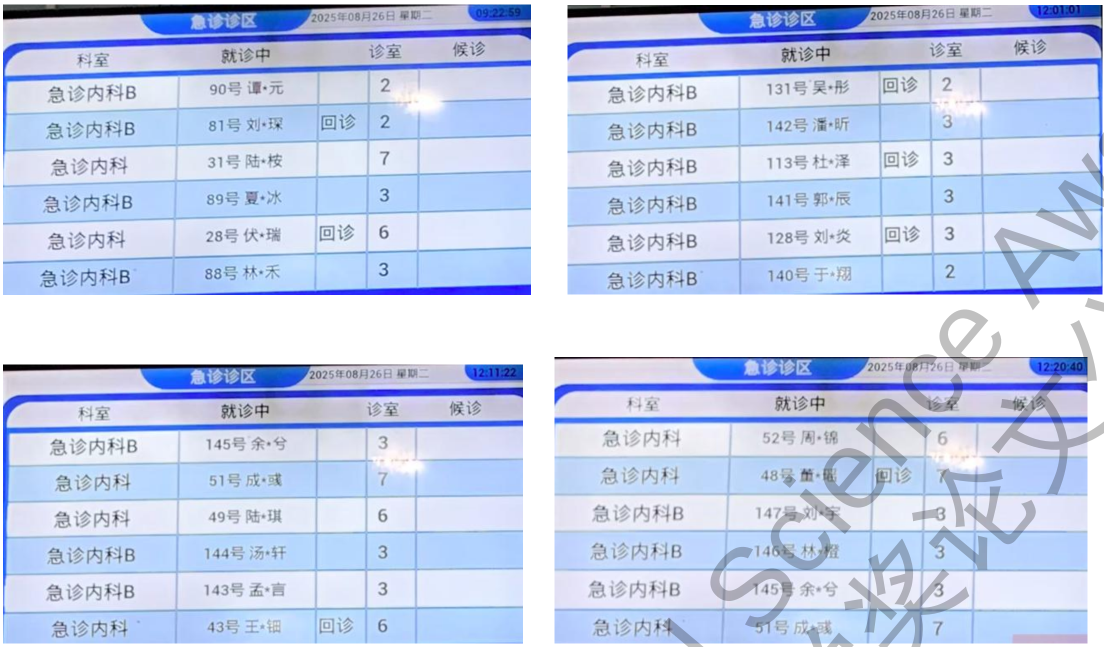


Fig. 5. The visiting fever patients are record on-site from 8:00 a.m. to 12:00 p.m. on July 16th, 2025.

Fig. 6. Spare clinics 8# - 16# to cope with the challenges of outbreak of epidemic or pandemic.

论文修改说明

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1、修改处 1

论文原题目： Optimizing physician scheduling based on dynamic mixed integer programming algorithm in fever clinics

修改后的论文题目： Mathematical modeling for physician scheduling in fever clinics based on dynamic mixed integer programming

题目修改说明： 把“Optimizing”优化修改为“Mathematical modeling”数学建模，更准确地概括了本论文的研究内容，论文内容主要建立最小化发热门诊医生与病人排队等待时间的数学模型。

2、**修改处 2**

对论文摘要进行了修改，加了“In this paper, the mathematical model for physician scheduling was established which consist of three parts as work time cost of on-duty physicians, work time cost of secondment physicians from other departments, and waiting time cost of fever patients (which is not found in other literatures).”

把本论文的数学建模加入了摘要中。

3、**修改处 3**

对论文结论进行了修改，加了“In this article, the objective function C3 consist of waiting time cost of fever patients which is not found in other literatures. The objective function C3 was minimized by applying Bellman equation to optimize state value function and optimal policy.).”

在结论中对本论文的创新点“ objective function C3”进行强调与其它文献不同之处，目前国内外文献中没查到在发热门诊中，通过最小化病人等待时间，从而减小病人交叉感染的风险，对医生科学排班。