Less Is More? Effort Allocation, Specialization, and Fixed Participation Costs in Multi-dimensional Score Contests for College Admissions

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Abstract

Many college admissions systems evaluate applicants along several academic or extracurricular dimensions, creating incentives for students to allocate limited preparation time across multiple tasks. In this paper, I model this setting as a multi-dimensional score contest in which applicants allocate costly preparation time across dimensions. Because score production exhibits diminishing marginal returns to time, an applicant may optimally devote positive effort to a lower-productivity dimension when scores from several dimensions are aggregated.

The baseline model contains two applicants, two dimensions, and two applicant types. Each type has a productivity advantage in one dimension. Aggregate scores enter a Tullock contest success function, while time is costly. I compare a multi-dimensional mechanism, under which both dimensions can contribute to the aggregate score, with a single-dimensional mechanism that restricts each applicant to the dimension corresponding to the applicant's comparative advantage. I find that, in the baseline with a square-root score technology and winning probabilities proportional to aggregate scores, each applicant's total productive effort is the same under both mechanisms, while effort is reallocated from the lower-productivity dimension to the higher-productivity dimension. Thus, the formal baseline result concerns the degree of specialization rather than an unconditional welfare ranking: the multi-dimensional allocation is itself cost-minimizing for a given aggregate score, and the multi-dimensional mechanism induces a higher equilibrium aggregate score for each applicant under the assumed technology.

Moreover, if each active dimension carries an additional fixed participation cost(f>0), a cost-accounting comparison of the baseline equilibrium allocations shows that the single-dimensional mechanism reduces each applicant's total private cost by exactly f. The paper also discusses, without formally modeling, how greater specialization may generate additional non-score value through deeper field-specific investment. The equality of total productive effort also holds in three separate extensions that vary the production exponent, the Tullock exponent, or the size of the symmetric applicant pool one at a time. These extensions show that production technology, contest sensitivity, and applicant pool size affect the level of equilibrium effort, while the mechanisms continue to differ in how effort is allocated across dimensions, except under linear score production, when their allocations coincide. The policy discussion therefore distinguishes dimensions used to establish minimum competence from dimensions used for continued competitive differentiation.

JEL Classification: C72, D74, I23

Keywords: college admissions, Tullock contest, multi-dimensional effort, specialization, fixed participation costs

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1 Introduction

Selective college admissions require applicants to allocate scarce preparation time among activities that contribute to several observable application components.

In the United States, many institutions consider academic preparation together with essays, recommendations, activities, awards, and other contextual information. For example, the University of California's first-year application asks applicants to classify activities and awards into six categories, while explicitly advising applicants that it values "quality, not quantity" (University of California Admissions, 2026a); its comprehensive-review framework also considers specialized talents, achievements, leadership, and significant projects (University of California Admissions, 2026b). These institutional features motivate a model of competition across multiple score-producing dimensions, but they do not imply that every applicant must maximize every dimension.

In China, most students apply to universities through the "Gaokao," which evaluates performance in a combination of subjects. Under the "3+1+2" structure, students choose one of physics or history and two additional subjects, yielding twelve possible combinations rather than the two combinations in the traditional humanities–science division (Ministry of Education of the People's Republic of China, 2022). As of 2026, the new Gaokao had been implemented in 29 provincial-level regions and covered approximately 98 percent of candidates (Ministry of Education of the People's Republic of China, 2026). The U.S. and Chinese systems differ substantially, but both illustrate how selection rules can affect the allocation of limited study time across evaluated dimensions.

This paper asks three specific questions. First, does restricting each applicant to a single comparative-advantage dimension reduce total productive effort? Second, how does the restriction change the allocation of effort across dimensions? Third, how do fixed participation costs affect the comparison between mechanisms? These questions deliberately separate productive effort, fixed participation costs, and specialization from the possible non-score benefits of deeper field-specific investment. The paper does not treat these concepts as interchangeable measures of efficiency.

1.1 A motivating participation game

Before introducing endogenous effort, consider the following illustrative participation game. It is used only to motivate the possibility that competitive pressure induces broader participation; it is not the formal model of the paper. Suppose there are two students and two dimensions. If the number of dimensions in which they participate is the same, each wins with probability $1/2$. If one student participates in both dimensions while the other specializes, the broadly participating student wins

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with probability $\alpha > 1/2$. Let $f > 0$ be the fixed cost of entering one dimension and let $V > 0$ be the value of admission. The resulting payoff matrix is shown in table 1. In each cell, the first payoff belongs to the row applicant and the second to the column applicant.

Table 1: Illustrative participation game

	Opponent: two dimensions	Opponent: one dimension
Two dimensions	$\left(\frac{V}{2} - 2f,\ \frac{V}{2} - 2f\right)$	$(\alpha V - 2f,\ (1-\alpha)V - f)$
One dimension	$((1-\alpha)V - f,\ \alpha V - 2f)$	$\left(\frac{V}{2} - f,\ \frac{V}{2} - f\right)$

If $(\alpha - 1/2)V > f$, participating in both dimensions is a strictly dominant strategy, yet mutual participation in both dimensions gives each applicant a lower payoff than mutual specialization. This example has a prisoner's-dilemma structure, but effort and scores remain exogenous. The formal model therefore asks whether the specialization intuition survives when applicants choose effort strategically.

1.2 Model and main findings

I construct a game-theoretic model in which applicants choose time investments that generate scores. Each applicant has a productivity advantage in one of two dimensions. Scores are generated by a concave technology and are aggregated before entering a generalized Tullock contest success function. An applicant's payoff equals the expected admission value minus productive effort.

A key distinction is between two mechanisms. Under the multi-dimensional mechanism D_m, effort in either dimension can contribute to the applicant's aggregate score. The mechanism does not mechanically require strictly positive effort in every dimension; positive effort in both dimensions emerges endogenously when the score technology is strictly concave. Under the single-dimensional mechanism D_s, a Type A applicant is restricted to dimension A and a Type B applicant is restricted to dimension B. The baseline therefore assumes that the mechanism observes or correctly assigns comparative-advantage types; endogenous self-selection is outside the model.

In the baseline case with one applicant of each type, $\omega = 1/2$, and $r = 1$, each applicant exerts total productive effort $V/8$ under both mechanisms. Under D_m, a Type A applicant allocates

$$t_{iA}^{(m)*} = \frac{Vk^2}{8(k^2+1)}, \qquad t_{iB}^{(m)*} = \frac{V}{8(k^2+1)},$$

whereas under D_s the same applicant allocates $V/8$ to dimension A and zero to dimension B. The restriction therefore changes specialization but not total productive effort.

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This result does not, by itself, establish that the single-dimensional mechanism produces more output or higher welfare. For a fixed aggregate score, the multi-dimensional allocation minimizes the time required to produce that score; moreover, the equilibrium aggregate score is higher under D_m than under D_s in the baseline technology. The paper's formal comparison is consequently stated in terms of total productive effort and its allocation.

In a conditional accounting exercise that holds the baseline activation pattern fixed, assigning a fixed participation cost $f > 0$ to each active dimension yields total private costs of $V/8 + f$ under D_s and $V/8 + 2f$ under D_m. Hence, the single-dimensional mechanism has lower total private cost by exactly f under this additional assumption. The paper also discusses a possible implication of this greater specialization: deeper investment in an applicant's comparative-advantage dimension may generate additional value beyond the admissions score, such as deeper field-specific knowledge, research capability, or skill accumulation. Because these non-score benefits are not included in the baseline payoff function, this implication is presented as an interpretation rather than a theorem.

The extensions relax one baseline restriction at a time. With $\omega \in (0, 1]$, total effort is $\omega V/4$ under both mechanisms. With $r \in (0, 1]$ and $\omega = 1/2$, total effort is $rV/8$. With n symmetric applicants, total effort per applicant is $V(n-1)/(2n^2)$. These results preserve the equality of total productive effort while clarifying how production technology, contest sensitivity, and applicant pool size affect equilibrium effort.

1.3 Organization

Section 2 reviews the related contest literature and the education-economics literature on student time allocation. Section 3 defines the model and separates productive effort from fixed costs. Section 4 derives the baseline equilibria. Section 5 presents three extensions, with proofs in the appendix. Section 6 discusses applications to U.S. and Chinese college admissions and clearly labels claims that are not formally modeled. Section 7 concludes.

2 Related Literature

2.1 Contest theory and multi-dimensional competition

This paper builds on the Tullock contest framework (Tullock, 1967, 1980). In a Tullock contest, a participant's probability of winning depends on the participant's effort or score relative to the aggregate effort or score of all participants (Baye et al., 1994). Subsequent work studies equilibrium

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existence, uniqueness, heterogeneity, information, entry, risk attitudes, and prize design in such contests (Ryvkin, 2007; Ewerhart, 2015; Feng and Lu, 2017; Einy et al., 2017; Fu et al., 2016; Fu and Wu, 2018; Schweinzer and Segev, 2012).

The closest related work is by Lu et al. (2024), who study performance bundling in multi-dimensional competitions. They compare separate and bundled reward structures in competitions between two potentially asymmetric players and show that the desirability of bundling depends on the degree of player asymmetry. The present paper instead compares two restrictions on the set of score-producing dimensions and focuses on the resulting allocation of productive effort.

The paper is more distantly related to the literature on group contests (Nitzan, 1991a,b; Baik and Lee, 1997, 2007; Esteban and Ray, 1999, 2001; Skaperdas, 1998; Balart et al., 2017; Eliaz and Wu, 2018). Group-contest models study how several individuals jointly determine a group's success, whereas the present model studies how one individual allocates effort across several dimensions before an individual-level contest outcome is determined. The connection lies in the aggregation of multiple inputs, whereas the strategic unit differs.

2.2 Student time allocation and educational production

A second body of research treats student time and effort as scarce, endogenous inputs into educational achievement. Using detailed time-use data, Stinebrickner and Stinebrickner (2004) show that study time is an important input into college outcomes, while Stinebrickner and Stinebrickner (2003) examine the trade-off between schoolwork and employment and emphasize that work hours are chosen endogenously. Metcalfe et al. (2019) exploit major football tournaments to vary the opportunity value of leisure, and Phipps and Amaya (2023) show that an additional course burden can lower grade performance and increase failure. More recently, Barwick et al. (2026) estimate negative effects of mobile-app use—including peer-mediated use—on academic and early labor-market outcomes. Together, these studies motivate the paper's treatment of time as costly and of competing activities as opportunity costs.

The strategic nature of study time is especially explicit in Conley et al. (2024), who develop and estimate an equilibrium model in which students' study-time choices are affected by their social networks and in turn determine academic achievement. Their strategic interaction operates through peer behavior, whereas the present paper studies interaction through relative admission probabilities. The present model also differs by asking how a given applicant divides productive effort across several score-producing dimensions rather than choosing only aggregate study time.

A related education-production literature shows that the return to time varies across activities

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and environments. Fiorini and Keane (2014) study how children's allocation of time across activities affects cognitive and noncognitive development. Lavy (2015) and Rivkin and Schiman (2015) show that the achievement effect of instructional time varies with country context, the amount of instructional time, and classroom environments. De Fraja et al. (2010) model educational attainment as the joint product of effort by students, parents, and schools. These findings motivate allowing the productivity of time to differ across dimensions and allowing returns to time to be nonlinear, represented in the model by k and ω. They do not identify the particular power-function specification used here. They also limit the interpretation of the model: a dimension with lower productivity in generating the admission score need not be socially unproductive if it develops noncognitive, health, or other skills that are omitted from the payoff function.

2.3 Positioning of this paper

Taken together, the two literatures provide complementary foundations. The education-economics literature shows that time is scarce, endogenously allocated, and heterogeneous in its educational productivity; contest theory explains how a competitive selection rule changes private incentives. This paper combines these elements in a tractable aggregate-score contest and shows that changing the admissible score dimensions can leave total productive effort unchanged while reallocating that effort across dimensions. The two mechanisms are treated as exogenous institutional alternatives; the paper does not solve for an optimal mechanism. The analysis is theoretical: it does not empirically estimate the response of applicants to admissions reform, and it does not establish that the single-dimensional mechanism is superior in aggregate output, multi-dimensional skill formation, or welfare.

Official admissions materials are used only to motivate and interpret the model. They are not treated as evidence that the model's behavioral or welfare conclusions hold empirically.

3 Model

3.1 Players, mechanisms, and effort

There are $n = n_A + n_B$ applicants. Type A applicants have a productivity advantage in dimension A, and Type B applicants have a productivity advantage in dimension B. Let $i \in \{1, \ldots, n\}$ index applicants and let $d \in \{s, m\}$ index the mechanism. Applicants choose effort simultaneously under complete information; applicant types and all model parameters are common knowledge. The

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solution concept is a pure-strategy Nash equilibrium.

The model compares two mechanisms:

- Under the single-dimensional mechanism D_s, Type A applicants can invest only in dimension A and Type B applicants can invest only in dimension B.
- Under the multi-dimensional mechanism D_m, both dimensions can contribute to every applicant's aggregate score. The strategy set permits zero effort in either dimension; the model does not impose an exogenous positive minimum.

Let $t_{iA}^{(d)} \geq 0$ and $t_{iB}^{(d)} \geq 0$ denote applicant i's productive time investments in dimensions A and B. Thus, under D_m the strategy is a vector in $\mathbb{R}_+^2$, whereas under D_s only the type-appropriate component can be positive. Here $\mathbb{R}_+$ denotes the set of nonnegative real numbers. Total productive effort is

$$t_i^{(d)} = t_{iA}^{(d)} + t_{iB}^{(d)}. \tag{1}$$

Under D_s, $t_{iB}^{(s)} = 0$ for a Type A applicant and $t_{iA}^{(s)} = 0$ for a Type B applicant.

Table 2: Core notation

Symbol	Meaning
D_s, D_m	Single-dimensional and multi-dimensional mechanisms
$d \in \{s, m\}$	Mechanism index
n_A, n_B, n	Numbers of Type A applicants, Type B applicants, and total applicants
$\mathbb{R}_+$	Set of nonnegative real numbers
$t_{iA}^{(d)}, t_{iB}^{(d)}$	Applicant i's productive effort in dimensions A and B
$t_i^{(d)}$	Total productive effort, excluding fixed participation costs
$S_{iA}^{(d)}, S_{iB}^{(d)}$	Dimension-specific scores
$S_i^{(d)}$	Aggregate score $S_{iA}^{(d)} + S_{iB}^{(d)}$
$P_i^{(d)}$	Probability that applicant i receives the prize
$U_i^{(d)}$	Expected payoff of applicant i
$C_i^{(d)}$	Total private cost in the fixed-cost accounting exercise
$k > 1$	Productivity coefficient in an applicant's comparative-advantage dimension
$\omega \in (0, 1]$	Score-production exponent; smaller values imply stronger diminishing returns
$r \in (0, 1]$	Tullock exponent governing contest sensitivity
$V > 0$	Value of admission
$f > 0$	Fixed participation cost of activating one dimension, used only in the cost comparison

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3.2 Score production

Let $S_{iA}^{(d)}$ and $S_{iB}^{(d)}$ denote dimension-specific scores and let

$$S_i^{(d)} = S_{iA}^{(d)} + S_{iB}^{(d)} \tag{2}$$

denote the aggregate score. The additive aggregator assumes that dimension-specific scores have been normalized to a common cardinal scale. Let $k > 1$ be the productivity coefficient in the applicant's comparative-advantage dimension and let $\omega \in (0, 1]$ be the score-production exponent. For a Type A applicant,

$$S_i^{(d)} = k \left(t_{iA}^{(d)}\right)^{\omega} + \left(t_{iB}^{(d)}\right)^{\omega}, \tag{3}$$

and for a Type B applicant,

$$S_i^{(d)} = \left(t_{iA}^{(d)}\right)^{\omega} + k \left(t_{iB}^{(d)}\right)^{\omega}. \tag{4}$$

A larger ω means weaker diminishing marginal returns. The boundary $\omega = 0$ is excluded because it would not provide a well-defined increasing score technology at zero effort.

3.3 Contest success and utility

Applicant i's probability of winning under mechanism d is

$$P_i^{(d)} = \frac{\left(S_i^{(d)}\right)^r}{\sum_{j=1}^{n} \left(S_j^{(d)}\right)^r}, \qquad r \in (0, 1], \tag{5}$$

whenever at least one aggregate score is positive. If all aggregate scores are zero, each applicant wins with probability $1/n$. This convention makes the game well defined on the full strategy space. The parameter r measures the sensitivity of winning probabilities to score differences.

The payoff of applicant i is

$$U_i^{(d)} = P_i^{(d)} V - t_i^{(d)}, \tag{6}$$

where $V > 0$ is the common value of admission. The coefficient on $t_i^{(d)}$ is normalized to one, so one unit of productive effort has unit marginal cost. The contest has one probabilistic prize, so the model is a normalized one-prize benchmark, not a literal model of an entire university admissions system.

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3.4 Fixed participation costs and the scope of specialized-depth claims

The baseline effort variable $t_i^{(d)}$ excludes costs that do not directly produce scores. Call a dimension active when the applicant allocates strictly positive effort to it. As a separate accounting exercise after solving the effort game, assign each active dimension a fixed participation cost $f > 0$, such as registration, information search, or task-specific preparation. Then the total private cost for this activity profile is

$$C_i^{(s)} = t_i^{(s)} + f, \qquad C_i^{(m)} = t_i^{(m)} + 2f, \tag{7}$$

in the interior baseline equilibrium, in which one dimension is active under D_s and both dimensions are active under D_m. Because these activation counts are fixed in the baseline comparison, f does not alter the continuous effort choices; it only changes the accounting measure of total private cost.

Additional non-score benefits from specialized depth are not included in the baseline payoff function. Any claim that deeper investment in an applicant's comparative-advantage dimension generates value beyond the admissions score therefore requires an additional assumption and is treated as an interpretive implication rather than a baseline model result.

4 Main Results

This section studies the baseline case with one applicant of each type, $n_A = n_B = 1$, $\omega = 1/2$, and $r = 1$. Fixed participation costs are excluded from the equilibrium choice problem, so $t_i^{(d)}$ refers only to productive effort. A superscript $*$ denotes an equilibrium value.

4.1 Equilibrium characterizations

4.1.1 Multi-dimensional score contest

Proposition 4.1 (Multi-dimensional mechanism)**.** *Under D_m, the unique equilibrium effort allocation of a Type A applicant i is*

$$t_{iA}^{(m)*} = \frac{Vk^2}{8(k^2+1)}, \qquad t_{iB}^{(m)*} = \frac{V}{8(k^2+1)}. \tag{8}$$

For a Type B applicant j,

$$t_{jA}^{(m)*} = \frac{V}{8(k^2+1)}, \qquad t_{jB}^{(m)*} = \frac{Vk^2}{8(k^2+1)}. \tag{9}$$

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Each applicant's total equilibrium productive effort is

$$t_i^{(m)*} = t_j^{(m)*} = \frac{V}{8}. \tag{10}$$

Proof. Consider a Type A applicant i. To simplify notation within the proof, suppress the mechanism superscript. With $\omega = 1/2$,

$$S_{iA} = k\sqrt{t_{iA}}, \qquad S_{iB} = \sqrt{t_{iB}}, \qquad S_i = S_{iA} + S_{iB}.$$

Therefore,

$$t_{iA} = \frac{S_{iA}^2}{k^2}, \qquad t_{iB} = S_{iB}^2.$$

Because the winning probability depends on the dimension-specific scores only through their sum S_i, any best response must use the least costly allocation that produces its chosen aggregate score. For a given aggregate score S_i, the applicant therefore chooses the dimension-specific scores to minimize total productive effort:

$$\min_{S_{iA}\in[0,S_i]} \left\{ \frac{S_{iA}^2}{k^2} + (S_i - S_{iA})^2 \right\}.$$

The first-order condition is

$$\frac{2S_{iA}}{k^2} - 2(S_i - S_{iA}) = 0,$$

which implies

$$S_{iA} = k^2 S_{iB}, \qquad S_{iA} = \frac{k^2}{k^2+1} S_i, \qquad S_{iB} = \frac{1}{k^2+1} S_i. \tag{11}$$

The objective is strictly convex, so this allocation uniquely minimizes the time required to produce S_i. It follows that

$$t_{iA} = \frac{k^2 S_i^2}{(k^2+1)^2}, \qquad t_{iB} = \frac{S_i^2}{(k^2+1)^2}, \qquad t_i = \frac{S_i^2}{k^2+1}. \tag{12}$$

By symmetry, a Type B applicant j has the same aggregate-score cost function, $t_j = S_j^2/(k^2+1)$, with the two dimension-specific allocations reversed.

Any equilibrium has $S_i, S_j > 0$: the all-zero profile admits a profitable small deviation, and a positive-score applicant facing a zero score can reduce effort while remaining positive without changing the winning probability. Hence the equilibrium score choices are interior. Since $r = 1$,

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applicant i's utility is

$$U_i = \frac{S_i}{S_i + S_j} V - \frac{S_i^2}{k^2 + 1}.$$

The first-order condition is

$$\frac{S_j}{(S_i + S_j)^2} V = \frac{2S_i}{k^2 + 1}. \tag{13}$$

Similarly, applicant j satisfies

$$\frac{S_i}{(S_i + S_j)^2} V = \frac{2S_j}{k^2 + 1}. \tag{14}$$

The second derivatives are strictly negative:

$$\frac{\partial^2 U_i}{\partial S_i^2} = -\frac{2S_j V}{(S_i + S_j)^3} - \frac{2}{k^2 + 1} < 0, \qquad \frac{\partial^2 U_j}{\partial S_j^2} = -\frac{2S_i V}{(S_i + S_j)^3} - \frac{2}{k^2 + 1} < 0.$$

Because equilibrium scores are positive, dividing equation (13) by equation (14) yields $S_i = S_j$. Let the common equilibrium score be S_m^*. Substitution gives

$$\frac{V}{4S_m^*} = \frac{2S_m^*}{k^2 + 1},$$

so

$$S_m^* = \sqrt{\frac{V(k^2 + 1)}{8}}. \tag{15}$$

Using equations (11) and (12) gives

$$t_{iA}^{(m)*} = \frac{Vk^2}{8(k^2 + 1)}, \qquad t_{iB}^{(m)*} = \frac{V}{8(k^2 + 1)}, \qquad t_i^{(m)*} = \frac{V}{8}.$$

The Type B allocation follows by symmetry. □

4.1.2 Single-dimensional score contest

Proposition 4.2 (Single-dimensional mechanism)**.** *Under D_s, the unique equilibrium effort allocation of a Type A applicant i is*

$$t_{iA}^{(s)*} = \frac{V}{8}, \qquad t_{iB}^{(s)*} = 0. \tag{16}$$

For a Type B applicant j,

$$t_{jB}^{(s)*} = \frac{V}{8}, \qquad t_{jA}^{(s)*} = 0. \tag{17}$$

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Each applicant's total equilibrium productive effort is

$$t_i^{(s)*} = t_j^{(s)*} = \frac{V}{8}. \tag{18}$$

Proof. A Type A applicant invests only in dimension A, so

$$S_i = k\sqrt{t_i}, \qquad t_i = \frac{S_i^2}{k^2}.$$

The applicant's utility is

$$U_i = \frac{S_i}{S_i + S_j} V - \frac{S_i^2}{k^2},$$

and the first-order condition is

$$\frac{S_j}{(S_i + S_j)^2} V = \frac{2S_i}{k^2}. \tag{19}$$

The Type B applicant has the symmetric cost function and satisfies

$$\frac{S_i}{(S_i + S_j)^2} V = \frac{2S_j}{k^2}. \tag{20}$$

As under D_m, equilibrium scores are positive. The second derivatives are strictly negative. Therefore, the first-order conditions are sufficient, and dividing them yields $S_i = S_j$. Let the common score be S_s^*. Then

$$\frac{V}{4S_s^*} = \frac{2S_s^*}{k^2},$$

so

$$S_s^* = \sqrt{\frac{Vk^2}{8}}. \tag{21}$$

Consequently,

$$t_i^{(s)*} = \frac{(S_s^*)^2}{k^2} = \frac{V}{8}.$$

The dimension-specific allocations follow from the restrictions defining D_s. □

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4.2 Comparison of the mechanisms

Theorem 4.3 (Effort invariance and specialization)**.** *The equilibrium total productive effort of each applicant is identical under the two mechanisms:*

$$t_i^{(s)*} = t_i^{(m)*} = \frac{V}{8}. \tag{22}$$

For a Type A applicant, the effort invested in the lower-productivity dimension under D_m is exactly reallocated to the higher-productivity dimension under D_s:

$$t_{iA}^{(s)*} - t_{iA}^{(m)*} = \frac{V}{8(k^2+1)} = t_{iB}^{(m)*}. \tag{23}$$

The symmetric statement holds for a Type B applicant.

Proof. The equality of total effort follows directly from Propositions 4.1 and 4.2. The reallocation identity in (23) follows by subtracting (8) from (16). □

The intuition for effort invariance can be seen from the aggregate-score cost functions. After optimal within-applicant allocation, producing aggregate score S_i requires $t_i = S_i^2/(k^2+1)$ under D_m, whereas under D_s it requires $t_i = S_i^2/k^2$. Within each symmetric contest, these productivity differences amount to a common scaling of all applicants' scores. Because the Tullock contest success function depends on relative scores, such a common scaling does not affect winning probabilities, leaving the same incentives over total productive effort.

Theorem 4.3 implies that moving from D_m to D_s does not reduce productive effort in the baseline. It changes where that effort is invested. Under D_m, diminishing marginal returns make it optimal to use both dimensions even though $k > 1$. The lower-productivity investment is therefore an equilibrium response to the score technology, not evidence of irrational behavior.

The equilibrium scores also differ. From equations (15) and (21),

$$S_m^* = \sqrt{\frac{V(k^2+1)}{8}} > \sqrt{\frac{Vk^2}{8}} = S_s^*. \tag{24}$$

Thus, the multi-dimensional mechanism induces a higher equilibrium aggregate score under the assumed additive, concave technology. The baseline theorem should therefore be read as an effort-allocation result, not as a proof that the single-dimensional mechanism dominates in output or welfare.

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4.3 Fixed participation costs and specialized depth

Suppose that each active dimension carries a fixed participation cost $f > 0$. Combining equations (7) and (22) gives

$$C_i^{(s)*} = \frac{V}{8} + f, \qquad C_i^{(m)*} = \frac{V}{8} + 2f, \qquad C_i^{(m)*} - C_i^{(s)*} = f > 0. \tag{25}$$

Hence, conditional on the fixed-cost assumption, the single-dimensional mechanism reduces each applicant's total private cost by exactly f relative to D_m. This calculation evaluates the previously derived baseline equilibrium; it does not characterize an equilibrium of a game in which applicants can change the number of active dimensions in response to f.

The concentration of effort under D_s may also generate benefits that are not captured by the score-production technology itself. Let $B(t_{iA})$ denote an additional non-score value associated with the depth of investment in applicant i's comparative-advantage dimension. Assume that this non-score value increases strictly with field-specific effort. For a Type A applicant, equation (23) implies

$$B\left(t_{iA}^{(s)*}\right) > B\left(t_{iA}^{(m)*}\right). \tag{26}$$

The symmetric argument applies to a Type B applicant. This additional value may represent deeper field-specific knowledge, research capability, skill accumulation, or other benefits of sustained specialization that are not directly reflected in the admissions score. Because this additional value is not included in the baseline payoff function, this comparison is treated as an interpretive implication rather than an equilibrium result.

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Table 3: Status of the paper's main claims

Claim	**Included in baseline model?**	**Formally established?**
Equal total productive effort under D_s and D_m	Yes	Yes
Different allocation of effort across dimensions	Yes	Yes
Higher aggregate score per applicant under D_m in the baseline	Yes	Yes, from equilibrium formulas
Higher total cost under D_m when each active dimension costs f	Additional fixed-cost assumption	Conditional calculation
Additional non-score value from specialized depth under D_s	No	No; interpretive implication
Minimum-competence thresholds	No	No; discussed illustratively

5 Extensions

This section relaxes three baseline restrictions, one at a time: the production exponent ω, the contest-sensitivity parameter r, and the number of applicants $n_A + n_B$. The resulting equilibria are summarized in table 4; detailed derivations are provided in Appendix A.

Table 4: Summary of extensions

Extension	Parameters held fixed	Equilibrium effort per applicant	Main allocation implication
$\omega \in (0, 1]$	$r = 1, n_A = n_B = 1$	$\omega V/4$	Larger ω increases specialization under D_m
$r \in (0, 1]$	$\omega = 1/2, n_A = n_B = 1$	$rV/8$	r scales both dimension-specific efforts proportionally
$n = n_A + n_B \geq 2$	$\omega = 1/2, r = 1$	$V(n-1)/(2n^2)$	n changes the scale, not the baseline effort ratio

5.1 Alternative production technologies

In the baseline, $\omega = 1/2$. Now let $\omega \in (0, 1]$ while maintaining $r = 1$ and $n_A = n_B = 1$. A larger ω implies weaker diminishing marginal returns to effort.

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Proposition 5.1. *For $\omega \in (0, 1]$, each applicant's equilibrium total productive effort is identical under the two mechanisms:*

$$t_i^{(s)*} = t_i^{(m)*} = \frac{\omega V}{4}. \tag{27}$$

For $0 < \omega < 1$, a Type A applicant's dimension-specific efforts under D_m satisfy

$$\frac{t_{iA}^{(m)*}}{t_{iB}^{(m)*}} = k^{1/(1-\omega)}. \tag{28}$$

At $\omega = 1$, all productive effort is allocated to the comparative-advantage dimension even under D_m.

Proof. See Appendix A.1. □

The equality in (27) shows that effort invariance is not specific to the square-root technology. Total productive effort increases with ω because weaker diminishing returns increase the marginal return to continued investment. Equation (28) clarifies the allocation effect: as ω approaches one, the ratio of higher- to lower-productivity effort becomes increasingly large. When ω is smaller, the allocation is more balanced; when ω is larger, it is more specialized.

5.2 Alternative Tullock exponents

Maintain $\omega = 1/2$ and $n_A = n_B = 1$, and allow $r \in (0, 1]$. The probability that applicant i wins is

$$P_i = \frac{S_i^r}{S_i^r + S_j^r}.$$

Proposition 5.2. *For $r \in (0, 1]$, each applicant's equilibrium total productive effort is*

$$t_i^{(s)*} = t_i^{(m)*} = \frac{rV}{8}. \tag{29}$$

Under D_m, a Type A applicant chooses

$$t_{iA}^{(m)*} = \frac{rVk^2}{8(k^2+1)}, \qquad t_{iB}^{(m)*} = \frac{rV}{8(k^2+1)}. \tag{30}$$

Proof. See Appendix A.2. □

A larger r makes the contest success function more sensitive to score differences and therefore

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increases total effort. However,

$$\frac{t_{iA}^{(m)*}}{t_{iB}^{(m)*}} = k^2$$

is independent of r. Thus, r changes the intensity of competition but not the baseline degree of specialization.

5.3 Larger applicant pools

Maintain $\omega = 1/2$ and $r = 1$, and let n_A, n_B be positive integers, with $n = n_A + n_B \geq 2$.

Proposition 5.3. *In a symmetric equilibrium, each applicant's total productive effort is identical under the two mechanisms:*

$$t_i^{(s)*} = t_i^{(m)*} = \frac{V(n-1)}{2n^2}. \tag{31}$$

Under D_m, a Type A applicant chooses

$$t_{iA}^{(m)*} = \frac{k^2 V(n-1)}{2n^2(k^2+1)}, \quad t_{iB}^{(m)*} = \frac{V(n-1)}{2n^2(k^2+1)}. \tag{32}$$

Proof. See Appendix A.3. □

At the symmetric equilibrium, each applicant wins with probability $1/n$. For integer $n \geq 2$, the effort term $(n-1)/n^2$ decreases strictly whenever the pool gains another applicant. Therefore, a larger applicant pool reduces individual equilibrium effort in this benchmark. The ratio of efforts across dimensions remains k^2, so the pool size changes the scale rather than the composition of effort.

6 Discussion

The formal results provide a benchmark for interpreting admissions systems, but real institutions include thresholds, multiple seats, imperfect information, heterogeneous prizes, and constraints that are absent from the model. This section therefore distinguishes direct model implications from institutional interpretation.

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6.1 U.S. college admissions

U.S. college admissions systems often combine several application components. The University of California, for example, uses a comprehensive-review framework and asks first-year applicants to report activities and awards in six categories, but it also states that it values "quality, not quantity" (University of California Admissions, 2026a,b). Cornell's current policy requires first-year applicants to submit SAT or ACT scores and states that those scores are reviewed in context with GPA, course rigor, extracurricular engagement, essays, and recommendation letters (Cornell University Undergraduate Admissions, 2026).

These examples support the descriptive claim that admissions evaluation can be multi-dimensional. They do not establish that universities mechanically reward the number of dimensions, nor do they verify the equilibrium predictions of the model. The model instead isolates one incentive: when performance in several dimensions contributes continuously to an aggregate competitive score, an applicant may optimally invest in a lower-productivity dimension because of diminishing marginal returns in the higher-productivity dimension.

6.1.1 Illustrative minimum-competence thresholds

Actual admissions systems may require applicants to demonstrate minimum competence in some dimensions. Consider a Type A applicant whose lower-productivity dimension is B. In the unconstrained baseline equilibrium under D_m,

$$t_{iB}^{(m)*} = \frac{V}{8(k^2+1)}, \qquad S_{iB}^{(m)*} = \sqrt{\frac{V}{8(k^2+1)}}.$$

Suppose that, in addition to the institution's published minimum requirement, applicants perceive a competitive benchmark $\bar{S}_B$ in dimension B, reflecting the level they believe is necessary to remain competitive for admission. If $\bar{S}_B > S_{iB}^{(m)*}$, this perceived benchmark is binding relative to the unconstrained equilibrium and requires additional effort in the lower-productivity dimension. Then feasibility requires

$$t_{iB} = S_{iB}^2 \geq \bar{S}_B^2 > \frac{V}{8\left(k^2+1\right)} = t_{iB}^{(m)*}. \tag{33}$$

If instead $\bar{S}_B \leq S_{iB}^{(m)*}$, the benchmark is non-binding because the applicant's unconstrained equilibrium performance already satisfies it.

The first case is especially relevant when published minimum standards do not coincide with

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applicants' perceived competitive benchmarks. USC, for example, lists a recommended TOEFL iBT minimum of 100 under the pre-2026 scoring scale, while also stating that its most competitive applicants score significantly above the published minimum and reporting an average TOEFL iBT score above 111 among admitted students (University of Southern California Undergraduate Admission, 2026). This does not imply a hidden cutoff at 111. Rather, the gap illustrates how applicants may infer a competitive benchmark $\bar{S}_B$ above the institution's published proficiency benchmark. Equation (33) shows that meeting this higher perceived benchmark requires additional effort beyond the unconstrained equilibrium level.

A direct policy response is to convert such competence dimensions into genuine sufficiency thresholds rather than leaving them as continuously evaluated scores. For a dimension whose primary purpose is to verify minimum preparation, the institution could set an explicit threshold $\bar{S}_B$ and assign no additional admissions weight to performance above that level. Let $R_B(S_{iB})$ denote the admissions weight associated with applicant i's performance in dimension B. Under the proposed rule, the marginal admissions reward would satisfy

$$\frac{\partial R_B(S_{iB})}{\partial S_{iB}} = 0 \qquad \text{for } S_{iB} \geq \bar{S}_B. \tag{34}$$

Under the proposed rule, once an applicant reaches the threshold $\bar{S}_B$, further increases in that score would receive no additional admissions weight.

A stronger implementation would convert the underlying score into a binary indicator before substantive application review. Applicants below $\bar{S}_B$ would be classified as not satisfying the requirement, while applicants at or above $\bar{S}_B$ would simply be classified as satisfying it; admissions officers would not observe the exact score above the threshold. In this way, $\bar{S}_B$ becomes a credible stopping point rather than merely a published minimum around which applicants continue to compete. This design could reduce incentives to devote additional effort to a competence dimension once the institution's stated standard has already been met.

6.1.2 Depth rather than activity count

The previous subsection considers excessive competition within a single competence dimension after a sufficient level of performance has already been reached. A related incentive problem can arise across dimensions. When applicants perceive that demonstrating achievement in an additional activity area may improve their admissions prospects, diminishing marginal returns within existing dimensions can make it privately attractive to broaden their activity portfolios rather than continue

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deepening areas in which they have already invested substantial effort. The model therefore suggests a distinction between rewarding the quality and depth of achievement within a dimension and rewarding the mere addition of another dimension.

The activities section of the UC application illustrates this issue in practice. Applicants may report up to 20 activities and awards, although UC explicitly emphasizes "quality, not quantity" and indicates that applicants need not use all available spaces (University of California Admissions, 2026a). UC does not state that activity count itself is an admissions criterion. Nevertheless, when the marginal value of additional breadth is not explicitly defined, applicants may remain uncertain about whether a relatively short activity list places them at a competitive disadvantage. The baseline incentive offers a possible interpretation: if entering an additional activity area is perceived to generate a positive admissions return, diminishing marginal returns within existing dimensions can make broader participation privately attractive even when total productive effort does not increase.

A more concrete policy would be to distinguish between primary activities and contextual activities. Applicants could be asked to designate a limited number of primary activities for substantive extracurricular evaluation. These activities would receive the main competitive weight and would be assessed in greater depth according to factors such as the depth of engagement, duration of participation, the substance of the work performed, the applicant's responsibility and contribution, concrete outputs, and, where relevant, broader social impact.

Applicants could still report additional activities beyond these designated primary activities, but those activities would serve mainly to provide contextual information and a fuller picture of the applicant rather than receive additional admissions weight simply by increasing the number of activities listed. Under this design, the admissions system would preserve useful information about applicants' broader interests and experiences while making clear that activity count itself is not a source of competitive value. The intended effect is to shift incentives away from accumulating additional activities for breadth and toward deeper investment in a smaller number of meaningful activities.

6.1.3 Distinguishing competence from competitive differentiation

The two policy proposals above reflect a broader design principle: admissions systems could distinguish explicitly between dimensions used to establish sufficient preparation and dimensions used for continued competitive differentiation. If applicants cannot tell which role a particular dimension plays, they may rationally behave as if higher performance is always rewarded. A published minimum may then fail to provide a stopping point, while the possibility that additional

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activities matter may continue to encourage broader participation.

Colleges could therefore classify evaluation dimensions ex ante according to their institutional purpose. Competence dimensions would be used primarily to verify that an applicant has reached a sufficient level of preparation. Once the stated threshold is met, further performance in that dimension would receive no additional admissions weight. Differentiation dimensions, by contrast, would remain subject to substantive comparison because further improvement in those dimensions is intended to convey information that the institution values for selection.

This distinction should also be communicated clearly to applicants. Admissions offices could state explicitly which requirements operate as genuine thresholds and which components remain competitively evaluated. Such transparency would reduce the need for applicants to infer unofficial benchmarks from the profiles of previously admitted students or to assume that every additional score, activity, or credential carries positive marginal value.

The discussion therefore suggests a hybrid principle: some dimensions can be used to establish sufficient preparation, whereas competitive differentiation can place greater weight on dimensions that reveal specialized strengths. This principle is not an optimal-mechanism result of the present model; it is a policy interpretation of the distinction between minimum competence and continued marginal rewards.

The formal results establish effort invariance across D_s and D_m and greater specialization under D_s. Holding the baseline activation pattern fixed, introducing a fixed participation cost per active dimension implies that total private cost under D_s is lower than under D_m by exactly f. They do not establish an unconditional welfare ranking, imply that greater specialization is always socially preferable, or identify an empirically optimal admissions policy.

6.2 Gaokao and Chinese college admissions

China's Gaokao provides a comparatively standardized measure of academic preparation while also allowing subject choice. The "3+1+2" structure replaced two traditional humanities–science combinations with twelve possible combinations (Ministry of Education of the People's Republic of China, 2022). By 2026, the new Gaokao had been implemented in 29 provincial-level regions and covered approximately 98 percent of candidates (Ministry of Education of the People's Republic of China, 2026).

Greater subject choice can permit students to allocate more effort toward subjects connected to their strengths and intended fields. At the same time, compulsory subjects and major-specific subject requirements mean that the system remains multi-dimensional. Unlike a pure competence require-

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ment, however, core Gaokao subjects are deliberately used for continuous academic differentiation: higher scores continue to affect an applicant's relative position even after basic competence has been demonstrated. The threshold proposal in Section 6.1 therefore does not imply that such dimensions should be converted into pass–fail requirements. Whether a dimension should be thresholded or continuously rewarded depends on its institutional purpose.

The Strong Foundation Program combines Gaokao performance with university-level assessment. Under the Ministry of Education's framework, Gaokao performance must account for at least 85 percent of the composite admission score, with university assessment and comprehensive evaluation contributing the remainder (Ministry of Education of the People's Republic of China, 2020). This structure illustrates how broad academic evaluation and more field-specific differentiation can coexist within the same admissions system.

It should not, however, be described as a full implementation of the threshold mechanism discussed above: because Gaokao performance retains substantial continuous weight, students continue to benefit from higher general scores after demonstrating minimum competence. Instead, the example illustrates the broader principle developed in Section 6.1.3: different dimensions can serve different institutional purposes and therefore need not share the same reward structure.

7 Conclusion

This paper compares a single-dimensional mechanism with a multi-dimensional mechanism in a score contest in which applicants allocate costly productive effort. In the baseline, both mechanisms generate the same total productive effort, $V/8$, but they generate different allocations. The multi-dimensional mechanism induces positive effort in both dimensions because score production is strictly concave in the baseline, while the single-dimensional mechanism concentrates effort in the applicant's comparative-advantage dimension. The equality continues to hold in three separate extensions that vary the production exponent, the Tullock exponent, or the applicant pool size one at a time.

The formal result is therefore an invariance-and-specialization result, not an unconditional proof that fewer dimensions always improve efficiency. The multi-dimensional allocation minimizes the time required to produce a given applicant-level aggregate score, and it generates a higher equilibrium aggregate score for each applicant in the baseline. A cost advantage for the single-dimensional mechanism arises only after adding a fixed participation cost per active dimension. The possible non-score benefits of deeper field-specific investment remain outside the formal model.

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The analysis has several limitations. First, the baseline contains only two dimensions and one probabilistic prize. Second, comparative-advantage types are known to the mechanism under D_s rather than privately chosen or inferred. Third, applicants have complete information and choose effort rationally; social pressure, uncertainty, and behavioral biases are omitted. Fourth, the score technology is separable across dimensions and does not allow learning complementarities. Fifth, minimum requirements and perceived competitive benchmarks are discussed but not incorporated into the equilibrium model. Finally, the institutional discussion is illustrative rather than an empirical test of the theory.

Within these limits, the model clarifies a useful distinction: the admissible set of score-producing dimensions can change the composition of effort even when it does not change total productive effort. Any policy assessment must then specify whether the relevant objective is aggregate score, private cost, specialization, non-score benefits from deeper field-specific investment, or social welfare.

A Proofs for the Extensions

A.1 Proof for the production-technology extension

We first consider $0 < \omega < 1$.

Under D_m, a Type A applicant's dimension-specific scores are

$$S_{iA} = k(t_{iA})^{\omega}, \qquad S_{iB} = (t_{iB})^{\omega},$$

and the aggregate score is $S_i = S_{iA} + S_{iB}$. The time required to generate the two scores is

$$t_{iA} = \left(\frac{S_{iA}}{k}\right)^{1/\omega}, \qquad t_{iB} = S_{iB}^{1/\omega}.$$

For a fixed aggregate score S_i, the applicant solves

$$\min_{S_{iA}\in[0,S_i]} \left\{ \left(\frac{S_{iA}}{k}\right)^{1/\omega} + (S_i - S_{iA})^{1/\omega} \right\}.$$

The first-order condition is

$$\frac{1}{\omega k^{1/\omega}} S_{iA}^{1/\omega-1} = \frac{1}{\omega} S_{iB}^{1/\omega-1}.$$

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Equivalently,

$$S_{iB} = S_{iA} k^{1/(\omega-1)}. \tag{35}$$

Since $S_i = S_{iA} + S_{iB}$,

$$S_{iA} = \frac{S_i}{1 + k^{1/(\omega-1)}}, \qquad S_{iB} = \frac{S_i}{1 + k^{1/(1-\omega)}}. \tag{36}$$

The second derivative is positive because $1/\omega > 1$, so this allocation uniquely minimizes total time.

Define

$$\kappa_\omega = \left[\frac{1}{k\left(1 + k^{1/(\omega-1)}\right)}\right]^{1/\omega} + \left[\frac{1}{1 + k^{1/(1-\omega)}}\right]^{1/\omega}. \tag{37}$$

Then the minimum productive effort required to generate aggregate score S_i is

$$t_i = \kappa_\omega S_i^{1/\omega}. \tag{38}$$

A Type B applicant has the same aggregate-score cost function, with the dimension-specific allocation reversed.

With $r = 1$, the Type A applicant's utility is

$$U_i = \frac{S_i}{S_i + S_j} V - \kappa_\omega S_i^{1/\omega}.$$

The first-order condition is

$$\frac{S_j}{(S_i + S_j)^2} V = \frac{\kappa_\omega}{\omega} S_i^{1/\omega - 1}. \tag{39}$$

The Type B applicant satisfies the symmetric condition. Dividing the two conditions implies $S_i = S_j$. Let the common equilibrium score be S_m^*. Substitution into (39) gives

$$\frac{V}{4S_m^*} = \frac{\kappa_\omega}{\omega} (S_m^*)^{1/\omega - 1},$$

so

$$(S_m^*)^{1/\omega} = \frac{\omega V}{4\kappa_\omega}. \tag{40}$$

Using (38),

$$t_i^{(m)*} = \kappa_\omega (S_m^*)^{1/\omega} = \frac{\omega V}{4}.$$

Under D_s, a Type A applicant's score is $S_i = k(t_i)^\omega$, so

$$t_i = \left(\frac{S_i}{k}\right)^{1/\omega}.$$

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The first-order condition is

$$\frac{S_j}{(S_i+S_j)^2}V = \frac{1}{\omega k^{1/\omega}} S_i^{1/\omega-1}.$$

Symmetry implies a common equilibrium score S_s^* satisfying

$$\frac{V}{4S_s^*} = \frac{1}{\omega k^{1/\omega}} (S_s^*)^{1/\omega-1}.$$

Therefore,

$$(S_s^*)^{1/\omega} = \frac{\omega V}{4} k^{1/\omega},$$

and

$$t_i^{(s)*} = \left(\frac{S_s^*}{k}\right)^{1/\omega} = \frac{\omega V}{4}.$$

This proves effort invariance for $0 < \omega < 1$.

To obtain the effort ratio under D_m, use the marginal-score condition directly:

$$k\omega(t_{iA})^{\omega-1} = \omega(t_{iB})^{\omega-1}.$$

Hence

$$\frac{t_{iA}}{t_{iB}} = k^{1/(1-\omega)}.$$

Finally, let $\omega = 1$. A Type A applicant produces

$$S_i = kt_{iA} + t_{iB}.$$

Since $k > 1$, every unit of time produces more score in dimension A, so the cost-minimizing allocation sets $t_{iB} = 0$. The problem under D_m is then identical to that under D_s. The symmetric first-order condition gives $S^* = kV/4$ and total effort $V/4$, which equals $\omega V/4$ at $\omega = 1$. □

A.2 Proof for the contest-technology extension

Maintain $\omega = 1/2$ and let $r \in (0, 1]$. The cost-minimizing allocation of a fixed aggregate score under D_m is unchanged from the baseline:

$$t_i = \frac{S_i^2}{k^2+1}.$$

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The Type A applicant's utility is

$$U_i = \frac{S_i^r}{S_i^r + S_j^r} V - \frac{S_i^2}{k^2 + 1}.$$

The first-order condition is

$$\frac{r S_i^{r-1} S_j^r}{(S_i^r + S_j^r)^2} V = \frac{2S_i}{k^2 + 1}. \tag{41}$$

The Type B applicant satisfies the symmetric condition. Dividing the conditions gives $S_i = S_j$. Let the common equilibrium score be S_m^*. Then

$$\frac{rV}{4S_m^*} = \frac{2S_m^*}{k^2 + 1},$$

so

$$(S_m^*)^2 = \frac{rV(k^2 + 1)}{8}.$$

Therefore,

$$t_i^{(m)*} = \frac{(S_m^*)^2}{k^2 + 1} = \frac{rV}{8}.$$

Using the score shares from (11),

$$t_{iA}^{(m)*} = \frac{rVk^2}{8(k^2 + 1)}, \qquad t_{iB}^{(m)*} = \frac{rV}{8(k^2 + 1)}.$$

Under D_s, $t_i = S_i^2/k^2$. The symmetric first-order condition gives

$$\frac{rV}{4S_s^*} = \frac{2S_s^*}{k^2},$$

so $(S_s^*)^2 = rVk^2/8$ and

$$t_i^{(s)*} = \frac{(S_s^*)^2}{k^2} = \frac{rV}{8}.$$

Thus $t_i^{(s)*} = t_i^{(m)*} = rV/8$. □

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A.3 Proof for the many-applicant extension

Maintain $\omega = 1/2$ and $r = 1$, and let $n = n_A + n_B \geq 2$. Under D_m, every applicant has the aggregate-score cost function

$$t_\ell = \frac{S_\ell^2}{k^2+1}.$$

Let

$$\mathcal{S} = \sum_{q=1}^{n} S_q.$$

Applicant ℓ's utility is

$$U_\ell = \frac{S_\ell}{\mathcal{S}} V - \frac{S_\ell^2}{k^2+1}.$$

The first-order condition is

$$V\frac{\mathcal{S}-S_\ell}{\mathcal{S}^2} = \frac{2S_\ell}{k^2+1}. \tag{42}$$

Because all applicants have the same aggregate-score cost function, consider a symmetric equilibrium with $S_\ell = S_m^*$ for all ℓ, so $\mathcal{S} = nS_m^*$. Substitution into (42) gives

$$\frac{V(n-1)}{n^2 S_m^*} = \frac{2S_m^*}{k^2+1}.$$

Hence

$$(S_m^*)^2 = \frac{V(n-1)(k^2+1)}{2n^2}$$

and

$$t_\ell^{(m)*} = \frac{(S_m^*)^2}{k^2+1} = \frac{V(n-1)}{2n^2}.$$

The dimension-specific allocation follows from (11):

$$t_{iA}^{(m)*} = \frac{k^2 V(n-1)}{2n^2(k^2+1)}, \qquad t_{iB}^{(m)*} = \frac{V(n-1)}{2n^2(k^2+1)}.$$

Under D_s, each applicant has cost $t_\ell = S_\ell^2/k^2$. The symmetric first-order condition becomes

$$\frac{V(n-1)}{n^2 S_s^*} = \frac{2S_s^*}{k^2}.$$

Thus

$$(S_s^*)^2 = \frac{k^2 V(n-1)}{2n^2}$$

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and

$$t_\ell^{(s)*} = \frac{(S_s^*)^2}{k^2} = \frac{V(n-1)}{2n^2}.$$

Therefore $t_\ell^{(s)*} = t_\ell^{(m)*}$. □

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致谢、研究过程与人工智能使用说明

选题来源与研究背景

本研究的选题来源于我在高中阶段同时参与学科竞赛、标准化考试和大学申请准备时产生的一个实际疑问：当学生需要在多个被评价的维度之间分配有限时间时，更多的评价维度是否一定能够带来更好的选拔结果？在与指导老师讨论后，我将这一问题抽象为多维竞争下的努力配置问题，研究不同计分维度如何影响总生产性努力、努力配置及固定参与成本。

论文最初主要关注多维竞赛导致的时间分散。在重新检查均衡结果后，我将核心结论收窄为“总生产性努力不变而努力配置发生变化”，并将固定参与成本、专业化深度可能产生的额外非分数收益和最低能力门槛分别作为附加假设或讨论内容处理。

学生与指导老师的分工

本研究的主体工作由学生本人完成。学生负责研究问题的提出与限定、文献阅读、模型设定、公式推导与均衡计算、正文撰写和参考文献整理，并对最终稿中的模型、证明、引用及文字表述进行复核。

指导老师为邝仲弘老师。指导老师主要在研究规范、模型表述、文献定位和论文结构方面提供建议，并对论文草稿提出修改意见。具体而言，指导老师帮助我区分模型中的正式结论、附加假设和政策解释，指出部分原有表述存在的过度推断问题，并建议将部分重复推导移至附录、补充教育经济学中关于学生时间配置的相关文献，以及进一步明确固定参与成本、专业化深度可能产生的额外非分数收益和最低能力门槛并非同一概念。上述指导为无偿学术指导。所有模型设定、公式推导、证明和最终文字均由学生本人重新理解并核验，最终提交版本由学生本人负责。

研究过程中遇到的困难及解决过程

研究过程中最主要的困难来自三个方面。第一，早期版本将“多维竞争导致努力分散”直接解释为效率损失，但在重新求解模型后，我发现，在基准设定下，单维机制与多维机制实际上产生相同的总生产性努力。因此，我重新区分了努力总量、努力配置、固定参与成本、产出以及专业化深度可能产生的额外非分数收益等不同概念，并将论文的核心结论调整为努力总量不变而努力配置发生变化。

第二，在扩展模型时，我需要分别分析生产函数凹度参数、Tullock竞争敏感度以及参与人数的变化。为避免代数结果与经济解释不一致，我重新推导了各项扩展中的均衡，并以

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$n = 2$ 等基准情形进行回代，核对不同参数对努力总量和配置的影响。

第三，将理论模型与现实大学录取制度联系起来时，容易把制度事实误写成模型已经证明的结论。为解决这一问题，我重新查阅大学及教育主管部门的官方材料，并在最终版本中明确区分“用于说明现实背景的制度事实”“模型中的正式结果”和“基于模型得到的政策解释”。例如，最低能力门槛只作为讨论中的说明性扩展，本文并未求解加入该约束后的完整均衡；专业化深度可能产生的额外非分数收益也未被纳入基准效用函数，因此仅作为可能的外部效应讨论。本研究为理论建模研究，不涉及实验、问卷调查或原始数据采集；论文中涉及的大学录取制度事实及相关文献均来自公开的官方资料和学术文献。

人工智能使用说明

经指导老师知情并同意，本研究曾在两个阶段使用 OpenAI ChatGPT 作为辅助工具：2025年7月4日至9月15日（具体模型版本已无法完全确认），以及2026年8月24日至9月15日（使用 GPT-5.6 Sol 和 GPT-6 Astra）。人工智能主要用于文献与公开资料的检索线索、英文语言修改与术语一致性检查、论文结构与重复内容检查、LaTeX 排版与交叉引用检查、符号一致性和证明表述检查，以及对学生已经完成的公式推导进行辅助性代数核对；相关输出均由学生本人重新核验后决定是否采用。论文最终提交版本中的所有模型设定、数学结论、引用和文字内容均由学生本人负责。

人工智能辅助主要集中在2026年的论文修改阶段，该阶段采用较高频率的连续多轮对话；2025年的初稿阶段则为间歇使用。累计使用时长约为10小时。按照赛事关于人工智能使用披露的要求，与本研究相关的人工智能聊天记录将按规定另行提交，以便核查研究和修改过程。

致谢

最后，我要感谢指导老师邝仲弘老师在本研究过程中给予的指导。邝老师在博弈论基础、文献阅读、模型表达和论文规范方面提出了许多具体建议，并帮助我认识到理论论文中必须严格区分假设、正式结论和现实解释。这些意见促使我重新检查原有推导、收窄论文结论，并更加认真地理解每一个公式和证明背后的经济学含义。

同时，我也感谢在学习和研究过程中为我提供课程、资料和讨论机会的老师和同学。完成这篇论文的过程使我更加认识到，数学推导本身并不是研究的终点；一个结论只有在假设清楚、逻辑成立、证据能够支持且研究者本人能够独立解释的情况下，才具有真正的意义。

对于论文中可能仍然存在的错误和不足，责任由学生本人承担。

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